NON-LINEAR DYNAMIC ANALYSIS OF RC SLENDER STRUCTURES SUBJECTED TO WIND LOADING BASED ON EXPERIMENTAL DATA

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Abstract— The goal of this paper is to propose a non-linear dynamic model based on experimental data and NBR-6123 (ABNT, 1988) to accomplish a non-linear dynamic analysis of slender structures subjected to wind loading. Several tests were conducted do asses the effective stiffness of slender structures as function of the internal loads. Once we have one equation to represent the real stiffness we proceed the dynamic analysis. At first we compute the static response given by the mean wind speed. In this part of the problem we consider the concept of effective stiffness to represent the physical non-linearity of material and a P-Delta method to represent the geometrical non-linearity. Considering the final stiffness obtained in that P-Delta method we compute the dynamic response given by the floating wind speed, according to the discrete dynamic model given by NBR-6123. A 40 m RC telecommunication tower was analyzed and the results obtained were compared with those given by linear static and dynamic models. We conclude that the non-linear dynamic analysis proposed here leads to values of internal loads 15% larger than the traditional linear dynamic analysis and 50% larger than the static analysis.

Keywords— Non-linear dynamic analysis, wind loading, optimization, experimental data.

1 Introduction

The models proposed by Brazilian Code NBR-6123-87 [1] to accomplish a dynamic analysis of structures subjected wind loading are based on linear dynamic models. In RC structures where the effective stiffness changes continuously due to non-linear material behavior and the level of strength, linear models could not describe precisely the structure behavior. Computation of cross-sections properties, and consequently the displacements and internal loads, in slender RC structures subjected to wind loading is a very difficult task because as the loads change along time, cross-sections properties change too. Which stiffness considers? Wind speed is defined by two components, one is the mean wind speed and another is the floating wind speed. Mean wind speed applies on the structures static loads, while floating wind speed applies dynamic loading. The models given by NBR-6123-87 [1] are based on linear dynamic models, in other words, they consider a constant stiffness along time, what does not happen in practice.

In this work the authors analyze a pre-fabricated 40 m RC telecommunication tower (Fig. 1) similar to others erected at Minas Gerais and Espírito Santo states of Brazil. For the effects of the mean wind speed on structure, the authors consider a non-linear behavior. In this phase, a P-Delta effect will be considered on the structure. In each iteration, the effective stiffness is given by Brasil and Silva [4]. After this method converging, we initiate the computation of the dynamic effects of wind given by the floating wind speed. The authors consider that the structure vibrates around an equilibrium position. This position is that one gave by the last iteration of P-Delta method. Then, the natural modes and frequencies of vibration are computed considering the effective stiffness given by the last iteration of P-Delta method. Once the natural shapes and frequencies are known, the dynamic analysis can be done according to NBR-6123:1987. The sum of the static, given by P-Delta method with Brasil and Silva (2004) curves, and dynamic components, provided by the discrete dynamic model of NBR-6123:1987, gives structural behavior.

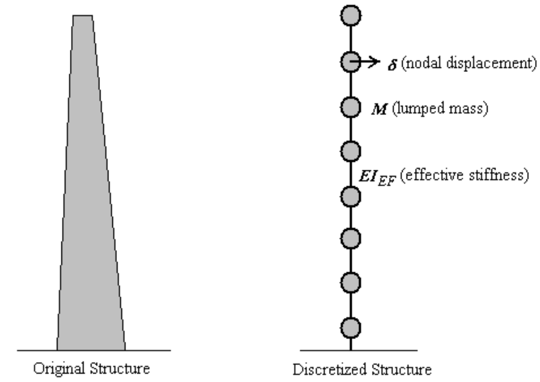

Figure 1. Typical RC telecommunication tower: original and discretized structure

2 Nonlinear dynamic analysis

2.1 Linear Static Analysis (LSA)

According to NBR-6123-87 [1] V_0 (meters per second) is the mean wind speed computed on 3 seconds, at 10 meters above ground, at a plain terrain with no roughness, with recurrence of 50 years. The topographic factor is S_1, while the terrain roughness is given by factor S_2, which is a function given by

$$S_2 = bF_r(z/10)^p \tag{1}$$

where b, p and Fr are factors which depend on the terrain characteristics, and z is the height above ground in meters. The statistic factor is S_3. Both S1, S2 e S3 are given by tables in Brazilian Code NBR-6123-87 [1]. The characteristic wind speed (meters per second) and the wind pressure (Pascal) are respectively

$$V_k = V_0 \cdot S_1 \cdot S_2 \cdot S_3, \quad q = 0.613 \cdot V_k^2. \tag{2}$$

The wind load (Newton) on an area A (projection on a vertical plane of a given object area in square meters) is computed as

$$F = q \cdot C_a \cdot A, \tag{3}$$

where C_a is the aerodynamical coefficient. The Brazilian Code NBR-6123:1987 presents tables for C_a values.

2.2 Linear Dynamic Analysis (LDA)

According to NBR-6123:1987, for the j-th degree of freedom, the total load X_j due to direct along wind is the sum of the mean and floating load given by:

$$X_j = \bar{X}_j + \hat{X}_j \tag{4}$$

where the mean load $\bar{X}_j$ is:

$$\bar{X}_j = \bar{q}_o b^2 C_j A_j \left(\frac{z_j}{z_r}\right)^{2p}, \tag{5}$$

being

$$\bar{q}_o = 0.613\bar{V}_p^2 \qquad \bar{V}_p = 0.69 V_0 S_1 S_3 \tag{6}$$

b and p indicated in Table 20 of NBR-6123:1987; z_r is the level of reference, equal to 10 meters in this work; $\bar{V}_p$ is design wind speed corresponding to the mean speed during 10 minutes at 10 meters above the ground level, for a terrain roughness (S_2) category II.

The floating component $\hat{X}_j$, is given by:

$$\hat{X}_j = F_H \psi_j \varphi_j \tag{7}$$

where

$$\psi_j = \frac{m_j}{m_o}, \qquad F_H = \bar{q}_o b^2 A_o \frac{\sum_{i=1}^{n} \beta_i \varphi_i}{\sum_{i=1}^{n} \psi_i \varphi_i^2} \xi$$

$$\beta_i = C_{ai} \frac{A_i}{A_o} \left(\frac{z_i}{z_r}\right)^p \tag{8}$$

being m_i, m_0, A_i, A_0, ξ and C_{ai}, respectively, the lumped mass at the i-th degree of freedom, a reference mass, the equivalent area at the i-th degree of freedom, a reference area, the dynamic amplification coefficient (Fig. 17 of NBR-6123:1987) and the area A_i aerodynamic coefficient.

Note that $\boldsymbol{\varphi} = [\varphi_i]$ is a given mode of vibration. To compute φ_i and ξ is necessary to consider the structure mass and stiffness. The lumped mass can be easily calculated by summing the mass around an influence region of the node. The total homogenized moment of inertia of the cross-section is given by

$$I_{\text{total}} = I_c + I_{s\,\text{hom}}, \qquad I_{s\,\text{hom}} = I_s\left(\frac{E_s}{E_{c\,\text{sec}}} - 1\right)$$

$$E_{c\,\text{sec}} = 0.9 \times 6600\sqrt{f_{ck} + 3.5} \quad (MPa), \tag{9}$$

being E_s, $E_{c\ \text{sec}}$, I_s, $I_{s\ \text{hom}}$, I_c and f_{ck}, respectively, the elasticity modulus of steel, the secant elasticity modulus of concrete (NBR-6118:1978), moment of inertia related to the structure axis of the total longitudinal steel area, the homogenized moment of inertia of the longitudinal steel area, the moment of inertia of the total cross-section area and the characteristic compressive resistance in *MPa* at 28 days old concrete. Since this model is based on linear dynamic models, we consider the cross-section moment of inertia the total stiffness, such as:

$$I = I_{\text{total}}, \tag{10}$$

of each section to compute stiffness matrix of the structure. This assumption may be justified because if this is a linear elastic model, any cross-section damage can be considered in this analysis, so the stiffness to be considered must be the total stiffness.

When r modes are considered in the analysis, the combination of these modes, for a given dynamic variable $\hat{Q}$, is computed as

$$\hat{Q} = \left[\sum_{k=1}^{r} \hat{Q}_i^2\right]^{1/2} \qquad Y_i = \frac{1}{3} X_i \tag{11}$$

is transversal dynamic load.

2.3 Non-Linear Dynamic Analysis (NDA)

As stated before, the loads due to the wind speed present two components, the static loads due to mean wind speed and the dynamic loads due to the floating wind speed. The static loads are computed as given in Eq. (5) and (6). We call the first results obtained using these equations as the first order static internal loads. At this point, we consider that the structure under those static loads is subjected to the P-Delta Effect. The static displacements ($\bar{\delta}_{i(j)}$), at the i-th node and the j-th iteration of the P-Delta method, are computed considering the effective stiffness. Differently of what occurs in Section 2.2, we consider the following expressions to compute the moment of inertia (Brasil and Silva [4]) at the i-th node and the j-th iteration of the P-Delta method:

$$I_{i(j)} = I_{EF\ i(j)} = w_{i(j)} I_{\text{total } i} \qquad w_{i(j)} = w\left(x_{i(j)}\right)$$
$$x_{i(j)} = \frac{\bar{M}_{k\,i(j-1)}}{M_{ui}}, \tag{12}$$

being I_{EF}, w, x, $\bar{M}_k$ and M_u, respectively, the effective moment of inertia, the parameter of effective stiffness, the level of strength, the working bending moment due to mean wind speed and the ultimate code based moment of a given cross-section. In Eq. (12) we consider that the damage occurred in the cross-sections is represented by the effective stiffness concept.

Finally, the P-Delta effect is computed, at the i-th node and the j-th iteration of the P-Delta method, as

$$\Delta\bar{M}_{k\,i(j)} = \Delta N_{k\,i}(\bar{\delta}_{i(j)} - \bar{\delta}_{i(j-1)})$$
$$\bar{M}_{k\,i(j)} = \bar{M}_{k\,i(j-1)} + \sum_{l} \Delta\bar{M}_{k\,l(j)} \tag{13}$$

We call the final results obtained using these equations as the second order static internal loads. Considering the stiffness obtained in final iteration of P-Delta method we compute the modes and frequencies of vibration of the structure and so accomplish the dynamic analysis, according to described by equations (7) and (8). We considered that the structure displaces around the equilibrium position given by the P-Delta Method.

3 The sample structure

The structure analyzed here is an RC telecommunication tower with 40 m long and diameter of 60 cm. The structure is cylindrical with cross-section in circular ring. Properties changes along the structure axis, because the thickness and steel area vary along the axis. The concrete used in the fabrication of the structure presents characteristic resistance (f_{ck}) at 28 days old equal to 45 MPa, what represents, according to Eq. (9) $E_{c\ sec}$ = 41.4 GPa. We consider the elasticity modulus of the structure $E = E_{c\ sec}$. The concrete covering is 25 mm. The concrete design resistance is f_{cd} = 45/1.3 MPa. The steel used in confection of structure presents f_{yd} = 500/1.15 MPa (steel design stress) and E_s = 210 GPa. The structure is discretized into 40 elements of one meter long each one. The properties are shown in Table 1.

In Table 1 we used the following notation: Node – the node number in the Finite Elements Method (FEM) Program; Height – level related to the ground level; Øext – external diameter of the cross-section; Thick. – thickness of the cross-section; M – nodal mass (lumped mass); A total – cross-section area; Ic – moment of inertia of the circular ring; nb – is the number of longitudinal bars of the reinforced concrete section; øb – diameter of longitudinal bars; As - – is the total longitudinal steel area; Rb – is the radius of the circle that pass along the longitudinal bars axis; Is - is the total moment of inertia of the steel area; Is hom - is the homogenized total moment of inertia of the steel area; Itotal - is the total homogenized moment of inertia of the reinforced concrete cross-section; Is/Itotal = ws – is the lower boundary value for w in each section.

According to NBR-6123-87 [1], we consider the basic wind speed of V_0 = 35 m/s, the topographic factor is S_1=1, terrain roughness category IV, class B, what gives $S_2 \equiv (b;\ p;\ Fr)$ and the statistic factor is S_3=1,1. As we stated before, the wind load on an area A is $F = q \cdot C_a \cdot A$, where C_a is the aerodynamical coefficients. Several equipment are installed on the structure, they are: stairway with anti-falls cable, platform with antennas supports, night signer lights, protection against atmospheric discharges system and installed antennas. The values of A and C_a are: tower, $0 \le z \le 40$ m, A = 0,6 m^2/m and C_a = 0,6; stairways, $0 \le z \le 40$ m, A = 0,05 m^2/m and C_a = 2; cables, $0 \le z \le 40$ m, A = 0,15 m^2/m and C_a = 1,2; platform and antennas supports, z= 40 m, A = 1 m^2 and C_a = 2; antennas, z= 40 m, A = 3 m^2 and C_a = 1. Table 2 shows the nodal mass and area for the structure analyzed.

Based on the results obtained by Brasil and Silva [4], in this section we adopt the following equation (Fig.2) for the effective stiffness parameters:

$$w = -1{,}5x^3 + 3{,}3x^2 - 2{,}5x + 1{,}1$$
$$w_s \le w \le 1, \quad \text{para} \quad i = 0, 1, \ldots, n \tag{14}$$

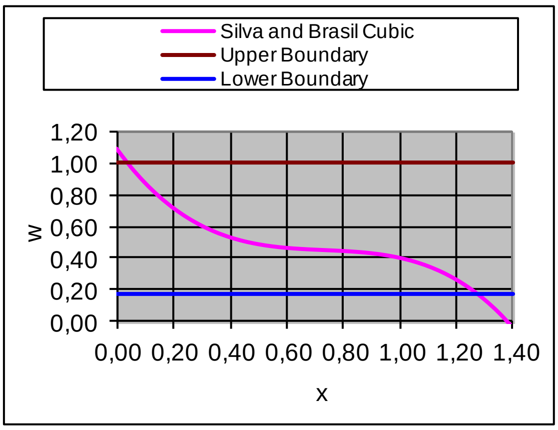

Figure. 2 – Effective stiffness adopted

Table 1. Structure Properties.

Node	Height (m)	ϕext (cm)	thick. (cm)	M (kgf)	A total (cm4)	Ic (cm4)	nb	øb (mm)	As (cm2)	Rb (cm)	Is (cm4)	Itotal (cm4)	Is/Itotal
1	40	60	10	802	1521	500417	20	13	25	27	8643	535650	7%
2	39	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
3	38	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
4	37	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
5	36	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
6	35	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
7	34	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
8	33	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
9	32	60	10	420	1521	500417	20	13	25	27	8643	535650	7%
10	31	60	13	531	1963	576678	20	13	25	27	8643	611911	6%
11	30	60	12	831	1850	560211	15	16	30	26	10483	602945	7%
12	29	60	11	473	1731	540542	15	16	30	26	10483	583275	7%
13	28	60	11	469	1716	537852	15	16	30	26	10483	580585	7%
14	27	60	11	469	1716	537852	15	16	30	26	10483	580585	7%
15	26	60	11	469	1716	537852	15	16	30	26	10483	580585	7%
16	25	60	11	469	1716	537852	16	16	32	26	11182	583434	8%
17	24	60	11	469	1716	537852	17	16	34	26	11881	586283	8%
18	23	60	11	469	1716	537852	18	16	36	26	12579	589132	9%
19	22	60	11	469	1716	537852	19	16	38	26	13278	591981	9%
20	21	60	11	469	1716	537852	20	16	40	26	13977	594830	10%
21	20	60	14	533	1973	578100	20	16	40	26	13977	635077	9%
22	19	60	15	896	2112	595395	15	20	47	26	16136	661174	10%
23	18	60	16	599	2238	608505	15	20	47	26	16136	674284	10%
24	17	60	13	520	1921	570729	16	20	50	26	17212	640894	11%
25	16	60	13	520	1921	570729	16	20	50	26	17212	640894	11%
26	15	60	13	520	1921	570729	17	20	53	26	18288	645279	12%
27	14	60	13	520	1921	570729	18	20	57	26	19364	649664	12%
28	13	60	13	520	1921	570729	19	20	60	26	20439	654050	13%
29	12	60	13	520	1921	570729	19	20	60	26	20439	654050	13%
30	11	60	13	520	1921	570729	20	20	63	26	21515	658435	13%
31	10	60	13	520	1921	570729	22	20	69	26	23667	667206	14%
32	9	60	16	599	2238	608505	22	20	69	26	23667	704981	14%
33	8	60	16	930	2249	609579	15	25	74	26	24744	710448	14%
34	7	60	17	605	2261	610622	15	25	74	26	24744	711491	14%
35	6	60	14	556	2063	589658	16	25	79	26	26394	697251	15%
36	5	60	14	556	2063	589658	16	25	79	26	26394	697251	15%
37	4	60	14	556	2063	589658	17	25	83	26	28043	703976	16%
38	3	60	14	556	2063	589658	17	25	83	26	28043	703976	16%
39	2	60	14	556	2063	589658	17	25	83	26	28043	703976	16%
40	1	60	18	628	2351	618137	17	25	83	26	28043	732455	16%
41	0	60	18	334	2351	618137	17	25	83	26	28043	732455	16%

Note that upper is equal 1,0 and lower values vary. Because of the safety coefficients adopted materials and design process, usually in tests structures present values of $x = M_k/M_u \geq 1,0$. For a 30 m structure tested by Brasil and Silva [4], the maximum value assumed by x was 1.33 and for other similar 40 m structure the maximum was $x = 1.53$.

Considering the lumped mass given in Table 1, the total homogenized moment of inertia for the LDA model and the effective moment of inertia of the final iteration of P-Delta Method for the NDA, we compute the natural modes and frequencies of vibration (Fig. 3). Note that in non-linear model the frequencies are smaller than in LDA. The coefficient of amplification ξ presented values until 2.35 for LDA and 2.65 for NDA.

The values obtained for bending moment in both models analyzed are shown in Fig. 4. In this figure we can see the following bending moments obtained:

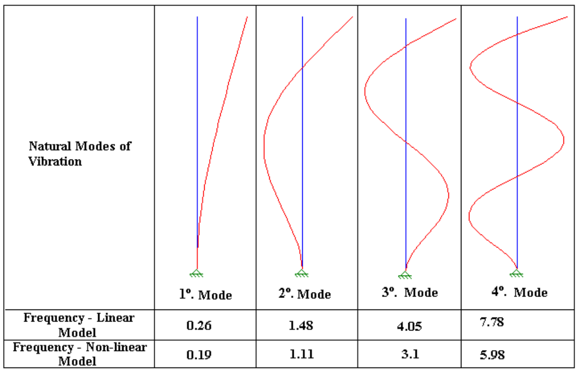

Figure 3 – Natural modes and frequencies (Hz) of vibration

M_{lsa} (LSA), M_{lda} (LDA), M_{nda} (NDA), M_d (design bending moment) and M_t (bending moment applied in tests). The LDA presented values of bending moment 1.3 times those given by the LSA, while the NDA presented values 1.5 times those from LSA. The design moments are 1.4 times those given by LSA and 1.1 times those given by LDA. Comparing the results we conclude that the design bending moment is satisfactory to the LDA, but is not satisfactory for NDA. Other important conclusion is about the excellent performance of the structure related to the safety coefficient near to the failure. The structure resisted a load around 1.53 times the design moment. As we stated before, this is due to the safety coefficients applied on material strength. Results from tests show that the structure resists satisfactorily the bending moments given by NDA.

Others important considerations here are related to the elasticity modulus of concrete. In this work we considered E = 41.4 *GPa*, computed according to NBR-6118:1978) Brazilian Code. This value is larger than values measured in tests, around 21 *GPa*, and larger than the value given by the revision of that Code, the new NBR-6118:2003, around 31.9 *GPa* for the adopted concrete. Tests showed that when we compute a certain function $w_1(x)$ considering a given elasticity modulus of concrete E_1 and solve the problem again using another value E_2, the new value of w is $w_2(x) = E_1w_1/E_2$, in other words, the quantity $E_1w_1 = E_2w_2 = E_iw_i$ is a constant for different values of E adopted.

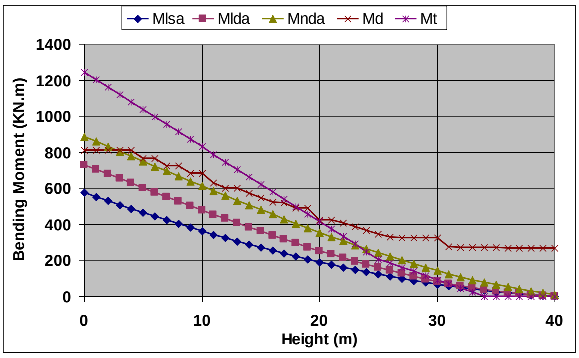

Fig. 4 – Bending moments

5 Conclusions

In this work we propose a Non-Linear Dynamic model based on experimental data and the discrete dynamic model given by NBR-6123-87 [1]. We adopted the effective stiffness concept to represent the physical non-linearity and used a P-Delta Method to compute the geometrical non-linearity. We considered a cubic equation to represent the effective stiffness. We accomplished the NDA considering the effective stiffness in function of the strength level in each iteration of the P-Delta Method. The effective stiffness obtained in final iteration of P-Delta Method was used to compute the natural frequencies and modes of vibration. We considered that the structure displaces around the equilibrium position given by P-Delta Method. Finally, we computed the sum of non-linear static and dynamic strength. We compared the values obtained from NDA with those from LSA and LDA. The LDA presented values of bending moment 1.3 times those given by the LSA, while the NDA presented values 1.5 times those from LSA. The design moments are 1.4 times those given by LSA and 1.1 times those given by LDA. We conclude that the design bending moment is satisfactory to the LDA, but is not satisfactory for NDA. Results from tests show that "in practice" the structure resists satisfactorily the bending moments given by NDA.

Suggestions for future works are: 1) process this structure considering different equations for the effective stiffness; 2) accomplish this NDA using the synthetic wind method (Franco [5]).

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