SEARCHING FOR ORBITS AROUND EUROPA THAT REQUIRES LOWER FUEL CONSUMPTION FOR STATIONKEEPING

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Abstract— Nowadays, there are several studies for missions that will place a satellite around Europa. There are many important aspects that deserve to be studied in this natural satellite of Jupiter. It makes the study of orbits around Europa a particular important part of the mission, since a good choice for them will reduce the costs related to station-keeping and then increasing the duration of the mission. In some previous studies, a search for frozen orbit around Europa was presented based in average techniques. The present research has the objective of using the new concept of stability of orbits with respect to station-keeping maneuvers that is available in the literature to study circular orbits around Europa. This concept is based in the integral of the perturbing forces over the time. This value can estimate the total variation of velocity required by the station-keeping propulsion system to compensate the perturbations suffered by the spacecraft. The value of this integral is a characteristic of the perturbations considered and the orbit chosen for the spacecraft. Numerical simulations are made showing the costs of station-keeping for circular orbits around Europa are shown as a function of the eccentricity and semi-major axis of the orbits.

Keywords— Astrodynamics, Stability of orbits, Artificial satellites, Planetary satellite, Station-keeping maneuvers.

1 INTRODUCTION

The problem of transfer maneuvers for spacecrafts is very important in space activities. There are many aspects to be considered, like the duration of the transfer, the fuel consumed, etc. The first important result was obtained by Hohmann, (1925), that found the solution for the transfer between two circular and coplanar orbits by applying the minimum variation of velocity to the spacecraft. Later, this type of maneuver that uses impulses as the control available was studied again by several researches. Some of them are Hoelker and Siller (1959), Jin and Melton (1991), Bender (1962), Eckel (1982), Gross and Prussing (1974), Prussing and Chiù (1986), Prado and Broucke (1995, 1996), Prado (1996) and Santos et al. (2012). Studies related to orbits of spacecrafts around Europa can be seen in Scheeres et al. (2001), Lara and Russell (2006), Paskowitz and Scheeres (2006), Lara (2008), Paskowitz and Scheeres (2009), Russell and Brinckerhoff (2009), Lara (2010) and Carvalho et al. (2012a, 2012b).

The goal of the present paper is to perform the integral over the time of all the perturbations for circular orbits around Europa, in order to evaluate the fuel consumption required to keep this orbit keplerian, so compensating the perturbations suffered by the spacecraft. This idea is explained in Prado (2013) and assumes that it is possible to use an ideal propulsion system to deliver a force that has the same magnitude, but in the opposite direction of the perturbations acting in the satellite. It is a kind of stability concept based in the total effects along the time that the perturbations cause in the motion of the spacecraft. So, this value
represents the total variation of velocity that the propulsion system needs to deliver. Applications studying the effects of the third-body perturbation of the Sun and the Moon are also shown in Prado (2013). To make a first study for orbits around Europa using this concept, only circular orbits will be considered, but with semi-major axis and inclinations free to assume any desired value. In this way, the results will allow mission designers to choose the best orbits that are suitable for their missions.

This new criterion of stability has several advantages related to the study of stationkeeping problems like the one shown here. Some of them are:

1) The orbits are always Keplerian, so this index can be evaluated for each perturbation independently of the others. In this way, it is possible to make a comparison of those values to choose which forces are important and needs to be considered in the dynamical model, depending on the accuracy desired by the mission;

2) This index depends on the force model used and on the orbit of the spacecraft;

3) It measures the total amount of variation of the velocity that comes from the perturbation forces considered in the dynamical model, so it also represents the consumption of fuel needed to maintain a Keplerian orbit for the spacecraft. In this way, this number shows which orbits require more fuel consumption for the stationkeeping maneuvers;

4) This index depends on the initial position of the spacecraft. So, in order to have a more accurate view, it is necessary to make an average over a certain number of orbits and not taken the result from only one orbit.

2 MATHEMATICAL MODELS

The mathematical model used here is now explained in detail. It is assumed that there is a spacecraft in a circular orbit around Europa. The forces acting in this spacecraft is the gravitational field of Europa, with the terms \( J_2 \), \( J_3 \) and \( C_{22} \), and the third-body perturbation due to Jupiter. Table 1 shows the numerical values of those constants. Note that the values are of the same order of magnitude, so it is necessary to take all of them into the model. The equations that describe the force field are also shown below.

2.1 FUNCTION FORCE DUE TO THE DISTURBING BODY

For the model considered in the present paper, it is necessary to calculate the terms \( R_2 \) of the disturbing function due to perturbation caused by a third-body in circular orbit (Jupiter is considered). The disturbing potential \( R_2 \) can be written in the form (Carvalho et al., 2010)

\[
R_2 = \frac{1}{2} N^2 r^2 (3 \cos^2(S) - 1) \tag{1}
\]

where \( r \) is the radius vector of the artificial satellite and \( N \) is mean motion of Jupiter. Here, \( S \) is the angle between the line that connects the massive central body and the perturbed body (the artificial satellite) and the line that connects the massive central body and the perturbing body (the third body). The artificial satellite is considered as a point mass particle in a circular orbit with osculating orbital elements: \( a = r \) (semi-major axis), \( i \) (inclination), \( g \) (argument of the periapsis), \( h \) (longitude of the ascending node). The numerical values for the physical parameters of Jupiter and Europa are shown in Table 2.

Using the relation between the angle \( S \) and the true anomaly \( f \) of the satellite we get (Broucke, 2003)

\[
\cos S = \alpha \cos f + \beta \sin f \tag{2}
\]

For the case of circular orbits, the products \( \alpha \) and \( \beta \) can be written in the form (Broucke, 2003; Prado, 2003)

\[
\alpha = \cos g \cos(h - M) - \cos i \sin g \sin(h - M),
\beta = -\sin g \cos(h - M) - \cos i \cos g \sin(h - M).
\tag{3}
\]

where \( M \) is the true anomaly of Jupiter. For the case of elliptical orbits alpha and beta are presented in Domingos et al. (2008) considering the planetary satellite in the same plane of the disturbing body, however when it is taken into account the inclination of the orbit of the disturbing body (in an elliptical orbit) alpha and beta are given in Liu et al. (2012). Now Eqs. (2) and (3) are replaced in Eq. 1. The disturbing potential

Table 1: Numerical values for \( J_2 \), \( J_3 \) and \( C_{22} \)

<table>
<thead>
<tr>
<th>Harmonics coefficients for the Europa (Lara and Russel, 2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_2 = 4.355 \times 10^{-4} )</td>
</tr>
<tr>
<td>( J_3 = 1.3784 \times 10^{-4} )</td>
</tr>
<tr>
<td>( C_{22} = 1.3065 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Table 2: Physical parameters

<table>
<thead>
<tr>
<th>Jupiter</th>
<th>Europa</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2.0477 \times 10^{-5} ) rad/s</td>
<td>( \mu_E = 3202.7 ) km(^3)/s(^2)</td>
</tr>
</tbody>
</table>
can be written as

\[
R_2 = -\frac{3}{8} a N^2 \left( \frac{1}{3} - c^2 - \frac{1}{2} (c - 1)^2 \cos(2g - 2h + 2M + 2f) - \frac{1}{2} (c + 1)^2 \cos(2g + 2h - 2M + 2f) - \cos(2h - 2M) - \cos(2g + 2f) + c^2 \cos(2g + 2f) + c^2 \cos(2h - 2M) \right)
\]

we will use the shortcut \( s = \sin i \) and \( e = \cos i \).

The potential at distance \( r \) from a mass and the force per unit mass there are related by (McCuskey, 1963)

\[
F = -\frac{\partial U}{\partial r}
\]

where \( U \) is the potential to be considered.

Replacing the Eq. (4) in Eq. (5) and using \( r = a, f = nt, M = Nt \), here \( n \) is mean motion of the satellite and \( t \) is the time, we get

\[
FR_2 = \frac{3}{8} a N^2 \left[ \frac{1}{3} - c^2 - \frac{1}{2} (c + 1)^2 \cos(2g + 2h - 2Nt + 2nt) - \frac{1}{2} (c - 1)^2 \cos(2g - 2h + 2Nt + 2nt) - \cos(2Nt - 2h) + c^2 \cos(2Nt - 2h) - \cos(2nt + 2g) + c^2 \cos(2nt + 2g) \right]
\]

### 2.2 Function Force Due to Non-Spherical Shape of Europa

To analyze the motion of a spacecraft around the planetary satellite it is necessary to take into account the Europa nonsphericity. As mentioned above, we only consider the harmonic coefficients \( J_2, J_3 \), and \( C_{22} \). Recalling that in this work we consider the satellite in a circular orbit around Europa. We get,

a) The zonal perturbation due to the oblateness \((J_2)\) is

\[
FJ_2 = -\frac{3a n^2}{2a} \left[ 1 - 3c^2 - 3 \cos(2nt + 2g) + 3c^2 \cos(2nt + 2g) \right]
\]

where \( c = J_2 R_E^2 \), \( R_E \) is the equatorial radius of Europa \((R_E = 1560.8 \text{ km})\).

b) The zonal perturbation due to the pear-shaped is defined by

\[
FJ_3 = -\frac{3a n^2}{2a} \left[ 15s^2 \sin(nt + g) - 5s^2 \sin(3nt + 3g) - 12 \sin(nt + g) \right]
\]

where \( s_1 = J_3 R_E^3 \).

c) For the sectorial perturbation we get

\[
FC_{22} = \frac{9n^2}{4a} \left[ (c - 1)^2 \cos(2nt + 2g - 2h) + (c + 1)^2 \cos(2nt + 2g + 2h) + 2 \cos(2h) - 2c^2 \cos(2h) \right]
\]

where \( \delta = C_{22} R_E^2 \).

### 2.3 Forces Involved in the Dinamics

The effects of the forces involved in the system given by the Eqs. (6), (7), (8) and (9) are to change the velocity \((V)\) of the satellite according to the physical law:

\[
\int_0^T F dt = \Delta V
\]

where \( F \) is the force by unit of mass, \( T \) is the period and \( F = FR_2 + FJ_2 + FJ_3 + FC_{22} \). We call the integral given in Eq. (10) by Perturbation Integral \((PI - \text{km/s})\),

\[
\int_0^T F dt = PI
\]

The Eq. (11) is numerically integrated to analyze the influence of the perturbations on the artificial satellite orbit around Europa.

### 3 Results

The main goal of the present paper is to show the evolution of the integral of the perturbing forces over the time for a spacecraft around Europa, as explained before.

![Figure 1: The minimum value for the Perturbing Integral (PI-km/s) as a function of the semi-major axis for polar orbits (i = 90°).](image-url)
value of the semi-major axis the gives the minimum value is near 2050 km for polar orbits. This minimum is about 30% smaller than the maximum value for the polar case, so a substantial savings in stationkeeping can be obtained by choosing the best altitude, if no other constraint of the mission imposes something different.

![Figure 2: Perturbation Integral (PI) of each force as a function of the time for orbits with semi-major axis of 1670 km, $g = 270^\circ$, $h = 90^\circ$ and $i = 90^\circ$.](image)

![Figure 3: Perturbation Integral (PI) of each force as a function of the time for orbits with semi-major axis of 2341 km, $g = 270^\circ$, $h = 90^\circ$ and $i = 90^\circ$.](image)

Fig. 2 and Fig. 3 show the evolution of the magnitude of the integral for all the forces considered in the dynamical model, individually. Studying the results in more detail, it is visible that the sum of the contributions of each individual effect is different from the total effect. This is explained by the fact that the perturbations have positive and negative signs with respect to each other. So, there are compensations, which mean that, for a given position of the spacecraft, some perturbing forces are trying to deviate the orbit from Keplerian, while some other forces are working in the opposite direction, so helping the control system to maintain the orbit keplerian and then reducing the values of the integral and the fuel consumption.

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The next interesting fact that can be explained is the variation of the Perturbing Integral with respect to the inclination of the orbit of the spacecraft. Fig. 2 shows a polar orbit with semi-major axis of 1670 km. Note that looking at the individual effects of each perturbation, some information can be obtained. The contribution of $J_3$ goes from near to 0.038. Regarding $J_2$, its contribution goes from near 0.1 again for the polar orbits. For the $C_{22}$ term, its contribution goes from near to 0.09. As a result of those effects, the total contribution of the potential of Europa goes closes to 0.14. It is interesting to note that the contribution of the $J_2$ term dominates the scene for the polar orbits. It is also necessary to take into account the compensations by having different signs of the forces. Polar orbits have more aligned effects of the individual terms of the potential of Europa. It is also clear that for polar orbits the effects of the total potential are larger than all the individual elements. It should also be noted that, for the polar orbits, the value of the integral for the total perturbations is smaller than the value obtained when considering only the potential of Europa. The third-body perturbation helps the control system for polar orbits, and this fact makes the costs smaller than the equivalent ones for equatorial orbits.

Looking now to Fig. 3, showing the results
for orbits with semi-major axis of 2341 km, it is clear that the third-body perturbation of Jupiter starts now to be the main force in the dynamics of the spacecraft. The potential of Europa now plays the role of working against the third-body perturbation, so reducing the value of the integral. But these compensating effects are not the same for all the orbits studied. The larger effects in the polar orbits make the value of the Perturbing Integral to decrease and the minimum of the graph is no longer sharp. Fig. 4 shows that the Perturbing Integral is almost constant from inclination from 70 to 110 degrees. So, this is one more fact to be considered when choosing an orbit for a spacecraft around Europa.

Figure 4: Perturbation Integral (PI-km/s) as a function of the inclination for orbits with semi-major axis of 2341 km

In Carvalho et al. (2013) was found frozen orbits for an artificial satellite around Europa taking into account the non-spherical shape ($J_2$, $J_3$) of the planetary satellite, and the perturbation of the third body (circular orbit). The orbit found presents the characteristics shown in Fig. 1 where the polar orbit has a lower value for the integral defined herein (PI). With the same approach developed in Carvalho et al. (2013), but now also taking into account the $C_{22}$ term, Fig. 5 was generated to analyze the behavior of polar orbits in the region shown in Fig. 1. Note that we found orbits that librate around the equilibrium point with varying amplitudes. The orbits with eccentricity 0.04 and 0.05 are frozen (Carvalho 2010, 2011), since they have smaller variation in this diagram.

A study was then made to know the effects of the main terms of the potential of Europa ($J_2$, $J_3$, $C_{22}$) and the third-body perturbation due to Jupiter. The results showed several interesting characteristics that depends on the orbit of the spacecraft, like: 1) the existence of orbits with minimum value for this index with respect to the semi-major axis, for a fixed inclination; 2) the existence of orbits with minimum value for this index with respect to the inclination, for a fixed semi-major axis; 3) the role of each individual term of the perturbing forces; 4) the forces have different signs with respect to each other, so, for a given position of the spacecraft, some forces are acting to destroy the keplerian orbit, while others are working together with the propulsion system to keep the keplerian orbit, so reducing the fuel consumption; 5) orbits with semi-major axis of 2341 km have a flat minimum, with values for the integral about the same for inclinations ranging from 50 to 90 degrees. With this new approach we find a region with values of semi-major axis, which can be found frozen orbits as shown in Fig 1 and Fig 5. The dynamic model presented here is very simplified, just to test this new concept to analyze the regions where frozen orbits can be found. In another paper we will use the criterion of integral of the force in a more realistic dynamics, considering elliptical orbits.

### 4 Conclusions

This paper studied the stability of circular orbits around Europa, using a new proposed index for this comparison, based on the integral of the perturbing forces over the time. This definition showed to be useful in the evaluation of the fuel consumption for station-keeping maneuvers.

Figure 5: Time evolution (300 days) of the eccentricity ($e$) and periapsis ($g$). (a) Initial conditions: $a = 2341$ km, $i = 90^\circ$, $g = 270^\circ$ and $h = 90^\circ$.

### acknowledgements

The author are grateful to FAPESP (Foundation to Support Research in São Paulo State) under the contracts N° 2011/05671-5 and 2012/21023-6, SP-Brazil, CNPq (National Council for Scientific and Technological Development) - Brazil for contracts 304700/2009-6, 3003070/2011-0 and CAPES.
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