CONTROL OF THE UNDERACTUATED SHIP - A FLATNESS BASED APPROACH

José Oniram de Aquino Limaverde Filho*, Eugênio Liborio Feitosa Fortaleza*

*GRACO - Grupo de Automação e Controle
Universidade de Brasília
Brasília, Distrito Federal, Brasil

Emails: joseoniram@ieee.org, efortaleza@unb.br

Abstract— Trajectory tracking is an issue of vital practical importance for manoeuvreing marine vessels and dynamic position system in many offshore oil field operations. In this paper, a flatness based approach is proposed on the tracking control design for a simplified underactuated ship model. The open loop trajectories are planned in an off-line manner exploiting the Liouvillian character of the nonlinear model. The Jacobian linearization of system around the planned trajectories will allow to obtain an incremental feedback control law to ensure a stabilization of the trajectory's tracking error. The performance of the tracking controller was evaluated by numerical simulations from arbitrary initial conditions and in the presence of external sinusoidal perturbations. The reference trajectory was chosen with respect to the Earth-fixed frame.

Keywords— Nonlinear Control; Trajectory Tracking; Differentially Flat Systems; Liouvillian Systems, Motion Planning

Keywords— Rastreamento de trajetória é uma questão de importância prática para manobramento de veículos marítimos e posicionamento dinâmico em muitas operações em campos de petróleo offshore. Neste trabalho, uma abordagem baseada na planejabilidade é proposta para o projeto de um controlador de rastreamento para um modelo subatuado simples de um navio. As trajetórias em malha aberta são planejadas de uma maneira off-line explorando o caráter Liouvilliano do modelo não-linear. A linearização Jacobiana do sistema em torno das trajetórias planejadas permitirá obter uma lei de controle incremental em malha fechada que garanta uma estabilização do erro de rastreamento de trajetória. A performance do controlador de rastreamento é avaliado por simulações numéricas a partir de condições iniciais arbitrárias e na presença de perturbações senoidais externas. A trajetória de referência foi escolhida com respeito ao eixo fixo da Terra.

Keywords— Controle Não-linear; Rastreamento de Trajetória; Sistemas Planejáveis; Sistemas Liouvillian, Planejamento do Movimento

1 Introduction

Control of mechanical systems is currently among one of the most active fields of research due to the diverse applications of mechanical systems in real-life. During the past century, a series of scientific, industrial, and military applications motivated rigorous analysis and control design for mechanical systems. On the other hand, the theoretically challenging nature of analysis of the behavior of non-linear dynamical systems attracted many mathematicians to study control systems (Olfati-Saber, 2001).

In this context, there is an important class of mechanical systems known as underactuated systems, which is defined to be one where the dimension of the space spanned by the control vector is less than the dimension of the configuration space (Toussaint et al., 2000). The possibility of controlling this systems is indeed appealing, for it allows us to reduce cost and weight as well as the occurrence of component failures (Reyhanoglu, 1996).

Belonging to the class of underactuated systems, control of ocean vessels, including ships and underwater vehicles, is an active field due to its theoretical challenges and important applications such as passenger and goods transportation, environmental surveying, undersea cable inspection, offshore oil installations, dynamic positioning, and many others (Pan and Do, 2009).

Tracking control of surface vessels has mainly been based on linear models, steering the same number of degrees of freedom as the number of control inputs available, and giving local results. In (Berge et al., 1999) and (Godhavn, 1996), output tracking control is discussed based on feedback linearization and Lyapunov theory. In (Behal et al., 2002) and (Pettersen and Nijmeijer, 1999), global practical tracking controllers are presented, where the tracking errors are made to converge to a neighborhood of the origin.

Commonly found in mobile robotics applications, the flatness property was proposed and developed by M. Fliess, J. Lévine, P. Martin e P. Rouchon (Fliess et al., 1992) in order to extend the theory of Kalman controllability for nonlinear cases (Ayadi, 2002). This property allows a complete parametrization of all systems variables (state, inputs, outputs) in terms of a finite set of independent variables, called the flat outputs, and a finite number of their time derivatives (Sira-Ramírez and Agrawal, 2004). For both planning and trajectory tracking problems, flatness is particularly advantageous for solving them.

Within this context, a special class of mechanical systems may exhibit the Liouvillian character, i.e. nonflat system with an identifiable flat subsystem of a dimension smaller than the dimension of the overall system. The Liouvillian character allows to compute the variables do not belong the flat subsystem as elementary integrations of the flat outputs and a finite number of their time derivatives. This class of systems has been introduced by Chelouah in (Chelouah, 1997).

For marine vehicles, in (Sira-Ramírez, 1999), the flatness property was used to design a dynamic feedback control scheme for a simplified nonlinear underactuated hovercraft model. A different flatness-based approach for the hovercraft was proposed in (Limaverde F. and Fortaleza, 2013). In (Sira-Ramírez and Ibanez, 2000), it is demonstrated that a nonlinear underactuated ship model has the Liouvillian character, which was used for an offline trajectory planning.

In this paper, following the results in (Sira-Ramírez and Ibanez, 2000), we proposed to design a time-varying tracking controller for a nonlinear underactuated ship model exploiting its Liouvillian character and the fact that Jacobian linearization of the system around nominal trajectories yields a flat system. The incremental flat outputs allows to design an incremental feedback control law that, when added to the nominal control, provides the final expression for the tracking controller, which will be applied directly to the nonlinear model.

The organization of this paper is as follows. In Section 2, the mathematical model describing the dynamics of a simplified underactuated ship model is introduced. Section 3 presents how the Liouvillian character of system allows an easy way to determine the nominal trajectories for all system variables. In this very section, it is shown the steps to derive the tracking controller. Section 4 presents some simulation results testing the robustness of the proposed controller. The conclusions and proposals for further research are presented in the last section.

2 The Underactuated Ship Model

In this section, the nonlinear model for the underactuated ship presented next was proposed in (Pettersen and Nijmeijer, 1999). The mathematical model of the system is given by the following set of differential equations:

$$\begin{aligned} \dot{x} &= u_1\cos(\psi) - v\sin(\psi) \\ \dot{y} &= u_1\sin(\psi) + v\cos(\psi) \\ \dot{\psi} &= u_2 \\ \dot{v} &= -\gamma u_1 u_2 - \beta v \end{aligned} \tag{1}$$

where x, y and ψ denote the position and the orientation of the ship in the Earth-fixed frame and v denotes the linear velocities in sway. The control inputs u_1 and u_2 represent the surge and yaw velocities respectively. The constants γ and β are strictly positive constants with $\gamma < 1$.

2.1 Off-line Trajectory Planning

Following the work in (Sira-Ramírez, 1999), from the System (1), it is possible to find a flat subsystem characterized by the sway velocity and angular orientation as flat outputs, which readily allows to compute the nominal control inputs from them and their time derivatives. Thus, if $v^*(t)$ and $\psi^*(t)$ are known, $u_1^*(t)$ and $u_2^*(t)$ are given by:

$$u_1^*(t) = -\frac{\dot{v}^*(t) + \beta v^*(t)}{\gamma\dot{\psi}^*(t)} \tag{2}$$

$$u_2^*(t) = \dot{\psi}^*(t) \tag{3}$$

The position variables can be expressed as quadratures of differential functions of the flat outputs, as seen below.

$$x^*(t) = \int\left\{-\frac{\dot{v}^*(t) + \beta v^*(t)}{\gamma\dot{\psi}^*(t)}\cos(\psi^*(t)) - v^*(t)\sin(\psi^*(t))\right\} \tag{4}$$

$$y^*(t) = \int\left\{-\frac{\dot{v}^*(t) + \beta v^*(t)}{\gamma\dot{\psi}^*(t)}\sin(\psi^*(t)) + v^*(t)\cos(\psi^*(t))\right\} \tag{5}$$

In many practical cases, it is easier to determine the nominal trajectories for the pair $(x^*(t), y^*(t))$ than for $(v^*(t), \psi^*(t))$. Therefore, after some algebraic manipulations, it is possible to obtain the following set of differential-algebraic equations:

$$\begin{aligned} \dot{\psi}^*(t) = &\left[\frac{1}{(\gamma-1)(\dot{x}^*(t)\cos(\psi^*(t)) + \dot{y}^*(t)\sin(\psi^*(t)))}\right] \\ &\times\{(\ddot{x}^*(t) + \beta\dot{x}^*(t))\sin(\psi^*(t)) \\ &- (\ddot{y}^*(t) + \beta\dot{y}^*(t))\cos(\psi^*(t))\} \end{aligned} \tag{6}$$

$$v^*(t) = -\dot{x}^*(t)\sin(\psi^*(t)) + \dot{y}^*(t)\cos(\psi^*(t))) \tag{7}$$

Without any loss of accuracy, a coordinate transformation (see (Pettersen and Egeland, 1996)) is used to replace trigonometric terms by simple polynomial equations, which is a global diffeomorphism:

$$\begin{aligned} z_1 &= x\cos(\psi) + y\sin(\psi) \\ z_2 &= -x\sin(\psi) + y\cos(\psi) \\ z_3 &= \psi \end{aligned} \tag{8}$$

This procedure will reduce the efforts to determine the incremental flat outputs after linearization around the nominal trajectories, as will be seen in the next section. The resulting model of the ship is then:

$$\begin{aligned} \dot{z}_1 &= u_1 + z_2 u_2 \\ \dot{z}_2 &= v - z_1 u_2 \\ \dot{z}_3 &= r \\ \dot{v} &= -\gamma u_1 u_2 - \beta v \end{aligned} \tag{9}$$

where the nominal trajectories for $z_1^*(t)$, $z_2^*(t)$ and $z_3^*(t)$ are directly obtained from Eq. 8. Therefore, the off-line computed trajectories will be used in a online feedback control scheme obtained from the following approximate linearization scheme.

3 A Trajectory Tracking Feedback Controller

Let us define the state variable tracking errors:

$$\begin{aligned} z_{1\delta} &= z_1 - z_1^*(t) \quad z_{2\delta} = z_2 - z_2^*(t) \\ z_{3\delta} &= z_3 - z_3^*(t) \quad v_\delta = v - v^*(t) \end{aligned} \tag{10}$$

The incremental control inputs $u_{1\delta}$ and $u_{2\delta}$ complement the nominal open loop control signals $u_1^*(t)$ and $u_2^*(t)$, respectively, to generate the tracking controllers:

$$u_1(t) = u_1^*(t) + u_{1\delta} \tag{11}$$

$$u_2(t) = u_2^*(t) + u_{2\delta} \tag{12}$$

with $u_1^*(t)$ and $u_2^*(t)$ being the open loop reference control inputs that would, ideally, steer the ship along the nominal trajectory.

A Jacobian linearization of the System (9), around the planned trajectories, is given by the following state representation:

$$\begin{aligned} \dot{z}_{1\delta} &= u_2^*(t) z_{2\delta} + u_{1\delta} + z_2^*(t) u_{2\delta} \\ \dot{z}_{2\delta} &= -u_2^*(t) z_{1\delta} + v_\delta - z_1^*(t) u_{2\delta} \\ \dot{z}_{3\delta} &= u_{2\delta} \\ \dot{v}_\delta &= -\gamma u_2^*(t) u_{1\delta} - \beta v_\delta - \gamma u_1^*(t) u_{2\delta} \end{aligned} \tag{13}$$

This linear time-varying system can be written in matrix form $\dot{\mathbf{X}}_\delta = \mathbf{A}(t)\mathbf{X}_\delta + \mathbf{B}(t)\mathbf{U}_\delta$, so we can also compute the Kalman's controllability matrix $\mathbf{C_K}(t)$ according to the formula developed in (Silverman and Meadows, 1967):

$$\mathbf{C_K}(t) = [\mathbf{B}(t),\ (\mathbf{A}(t) - \frac{d}{dt})\mathbf{B}(t),\ \cdots,\ (\mathbf{A}(t) - \frac{d}{dt})^{(n-1)}\mathbf{B}(t)] \tag{14}$$

It can be shown that System (13) loses controllability around the condition: $r^*(t) = 0$. Avoiding this singularity, the system is uniformly controllable, so we can extract the following full rank $n \times n$ matrix, $\mathbf{C}(t)$, from the Kalman's controllability matrix:

$$\mathbf{C}(t) = [\mathbf{b_1}(t),\ \cdots,\ (\mathbf{A}(t) - \frac{d}{dt})^{(\theta_1-1)}\mathbf{b_1}(t),\ \cdots,\ \mathbf{b_m}(t),\ \cdots,\ (\mathbf{A}(t) - \frac{d}{dt})^{(\theta_m-1)}\mathbf{b_m}(t)] \tag{15}$$

with θ_i, $i = 1, \cdots, m$, being the Kronecker controllability indices of the system, which must satisfy $\sum_i \theta_i = n$ (Poljak, 1990).

Under above assumptions, the incremental flat outputs can be computed by the following formula (Sira-Ramírez and Agrawal, 2004):

$$\mathbf{F}_\delta = \begin{bmatrix} F_{1\delta} \\ F_{2\delta} \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_m \end{bmatrix} \mathbf{C^{-1}}(t) X_\delta \tag{16}$$

where $\phi_\mathbf{j}, j = 1, \cdots, m$ are n-dimensional row vectors of the form

$$\phi_\mathbf{j} = [0, \cdots, 0, 1, 0, \cdots, 0] \tag{17}$$

with the 1 in the $(\sum_{i=1}^{j} \theta_i)$-th position.

Moreover, any uniformly non-zero, time-varying, scalar multiple of the flat output is also a flat output (Sira-Ramírez and Agrawal, 2004), which simplifies the expression for the flat output. Thus, choosing the controllability indices $\theta_1 = 2$ and $\theta_2 = 2$, and performing possibles simplifications, the incremental flat outputs of the linearized System (13) are given by:

$$F_{1\delta} = \alpha_1(t) z_{1\delta} + \alpha_2(t) z_{2\delta} + \alpha_3(t) z_{3\delta} + \alpha_4(t) v_\delta \tag{18}$$

$$F_{2\delta} = \eta_1(t) z_{1\delta} + \eta_2(t) z_{2\delta} + \eta_3(t) z_{3\delta} + \eta_4(t) v_\delta \tag{19}$$

where

$$\alpha_1(t) = \gamma u_2^*(t)[\gamma u_1^*(t) + \dot{z}_1^*(t) + u_2^*(t) z_2^*(t)] \tag{20}$$

$$\begin{aligned} \alpha_2(t) = \gamma(&-z_1^*(t) u_2^*(t)^2 - \dot{z}_2^*(t) u_2^*(t) \\ &+ \dot{u}_1^*(t) + \beta u_1^*(t)) \end{aligned} \tag{21}$$

$$\begin{aligned} \alpha_3(t) = \gamma\{&\dot{u}_1^*(t) z_1^*(t) + \gamma u_1^*(t)^2 + \beta u_1^*(t) z_1^*(t) \\ &+ (\dot{z}_1^*(t) - (\gamma - 1) u_1^*(t)) u_2^*(t) z_2^*(t) \\ &- u_2^*(t)^2 (z_1^*(t)^2 + z_2^*(t)^2) - \dot{z}_1^*(t) u_1^*(t) \\ &- \dot{z}_2^*(t) u_2^*(t) z_1^*(t)\} \end{aligned} \tag{22}$$

$$\alpha_4(t) = \gamma u_1^*(t) - \dot{z}_1^*(t) + u_2^*(t) z_2^*(t) \tag{23}$$

$$\eta_1(t) = -\gamma u_2^*(t)^2 (\gamma + 1) \tag{24}$$

$$\eta_2(t) = -\gamma(\dot{u}_2^*(t) + \beta u_2^*(t)) \tag{25}$$

$$\begin{aligned} \eta_3(t) = \gamma\{&(\gamma + 1) u_2^*(t)(u_2^*(t) z_2^*(t) - u_1^*(t)) \\ &- z_1^*(t)(\dot{u}_2^*(t) + \beta u_2^*(t))\} \end{aligned} \tag{26}$$

$$\eta_4(t) = -u_2^*(t)(\gamma + 1) \tag{27}$$

The controllability indices correspond to the number of times needed to derive the flat outputs in order to obtain the incremental control inputs in terms of the flats outputs and their time derivatives. Then, as the indices are equal to 2, we need to derive the Eq. (18) and Eq. (19) two times.

Furthermore, we can pre-compute the time-varying coefficients presents in this equations, which will be used as a strategy to determine the time derivatives of the flat outputs. Thus, it is easier to mount the matrix equation $\mathbf{F} = \mathbf{M}(t)\mathbf{X}$ with:

$$\mathbf{F} = \begin{bmatrix} F_{1\delta} & \dot{F}_{1\delta} & \ddot{F}_{1\delta} & F_{2\delta} & \dot{F}_{2\delta} & \ddot{F}_{2\delta} \end{bmatrix}^T \tag{28}$$

$$\mathbf{X} = \begin{bmatrix} z_{1\delta} & z_{2\delta} & z_{3\delta} & v_\delta & u_{1\delta} & u_{2\delta} \end{bmatrix}^T \tag{29}$$

where $\mathbf{M}(t)$ is square matrix (6×6) with time-varying coefficients.

Inverting the matrix $\mathbf{M}(t)$, we observe the differential parametrization of the linearized system (13) in terms of the incremental flat outputs and their time derivatives. Therefore, the controller design is greatly facilitated by the fact that the system is equivalent, under a time-varying state coordinate transformation and state feedback, to a controllable, time-invariant, multivariable linear system in Brunovsky's canonical form (Sira-Ramírez and Agrawal, 2004).

The incremental input controls can be easily obtained as follow:

$$\begin{aligned} u_{1\delta} &= C_1(t)F_{1\delta} + C_2(t)\dot{F}_{1\delta} + C_3(t)\ddot{F}_{1\delta} \\ &\quad + C_4(t)F_{2\delta} + C_5(t)\dot{F}_{2\delta} + C_6(t)\ddot{F}_{2\delta} \end{aligned} \tag{30}$$

$$\begin{aligned} u_{2\delta} &= C_7(t)F_{1\delta} + C_8(t)\dot{F}_{1\delta} + C_9(t)\ddot{F}_{1\delta} \\ &\quad + C_{10}(t)F_{2\delta} + C_{11}(t)\dot{F}_{2\delta} + C_{12}(t)\ddot{F}_{2\delta} \end{aligned} \tag{31}$$

where the time-varying coefficients in Eq. (30) and Eq. (31) are obtained by the last two of $\mathbf{M}^{-1}(t)$.

The linearized System (13) is therefore equivalent, under a change of input coordinates, to the following two decoupled linear systems in Brunovsky's form:

$$\ddot{F}_{1\delta} = \upsilon_1 \tag{32}$$

$$\ddot{F}_{2\delta} = \upsilon_2 \tag{33}$$

A dynamic tracking controller may be readily obtained by setting:

$$\upsilon_1 = -k_1\dot{F}_{1\delta} - k_0F_{1\delta} \tag{34}$$

$$\upsilon_2 = -k_1\dot{F}_{2\delta} - k_0F_{2\delta} \tag{35}$$

with k_0, k_1 chosen so that the closed loop characteristic polynomial $p(s) = s^2 + k_1s + k_0$ is a Hurwitz polynomial (Sira-Ramírez and Agrawal, 2004).

Since both $F_{1\delta}$ and $F_{2\delta}$ asymptotically converge to zero in an exponentially fashion, then, it follows the previous differential parametrization that, the state variable tracking errors also asymptotically converge to zero.

The final expression therefore for $u_1(t)$ and $u_2(t)$ are given by:

$$\begin{aligned} u_1(t) &= u_1^*(t) + \{C_1(t)F_{1\delta} + C_2(t)\dot{F}_{1\delta} \\ &\quad + C_3(t)[-k_1\dot{F}_{1\delta} - k_0F_{1\delta}] + C_4(t)F_{2\delta} \\ &\quad + C_5(t)\dot{F}_{2\delta} + C_6(t)[-k_1\dot{F}_{2\delta} - k_0F_{2\delta}]\} \end{aligned} \tag{36}$$

$$\begin{aligned} u_2(t) &= u_2^*(t) + \{C_7(t)F_{1\delta} + C_8(t)\dot{F}_{1\delta} \\ &\quad + C_9(t)[-k_1\dot{F}_{1\delta} - k_0F_{1\delta}] + C_{10}(t)F_{2\delta} \\ &\quad + C_{11}(t)\dot{F}_{2\delta} + C_{12}(t)[-k_1\dot{F}_{2\delta} - k_0F_{2\delta}]\} \end{aligned} \tag{37}$$

4 Simulation Results

Numerical simulations were carried out to asses the performance of the designed controllers in two cases. Following (Sira-Ramírez, 1999), we chose a typical circular trajectory defined in the earth fixed coordinate frame, of radius ρ, centered around the origin. The ship must follow in a counterclockwise sense in the plane (X,Y) with a given constant angular velocity of value ω. In the second simulation, for the same circular trajectory, we introduced in the non-actuated dynamics an unmodeled external sinusoidal perturbation, simulating a "wave field" effect.

The desired trajectory can be parametrized as follows:

$$x^*(t) = \rho\sin(\omega t) \tag{38}$$

$$y^*(t) = \rho(1 - \cos(\omega t)) \tag{39}$$

The corresponding nominal trajectories for the orientation angle and sway velocity were directly computed by Eq. (6) and Eq. (7). The open loop control signals $u_1^*(t)$ and $u_2^*(t)$ were obtained by Eq. (2) and Eq. (3). Moreover, it is possible to compute the trajectories nominal for $z_1^*(t)$, $z_2^*(t)$ and $z_3^*(t)$, as described in Eq. (8).

In the simulations, the initial ship position and orientation was taken to be:

$$\begin{aligned} x(0) &= 1 & y(0) &= -4 \\ \psi(0) &= 2\pi/3 & v(0) &= 0 \end{aligned} \tag{40}$$

The parameters of the reference trajectory and of the system model were set to be:

$$\rho = 5 \qquad \omega = 1 \qquad \beta = 1.5 \qquad \gamma = 0.5 \tag{41}$$

The control parameters were chosen as:

$$k_1 = 6 \qquad\qquad k_0 = 9 \tag{42}$$

For the external sinusoidal perturbation, we choose the following form:

$$\lambda(t) = 2.5[\sin(10t) + 0.5\cos(\pi 10t)] \tag{43}$$

In all numerical simulations, the ship's path is shown in red dotted line, while the desired trajectory is in blue. Furthermore, input saturation was added to the system.

Figure 1 depicts the ship motion in the plane (X,Y) for the initial conditions in Eq. (40). In Figure 2, we can see the corresponding position time evolution. The corresponding angular orientation and sway velocity are shown in Figure 3. The time evolution of the control inputs is shown in Figure 4. Figure 5 displays the result of the simulation in the presence of external sinusoidal perturbation.

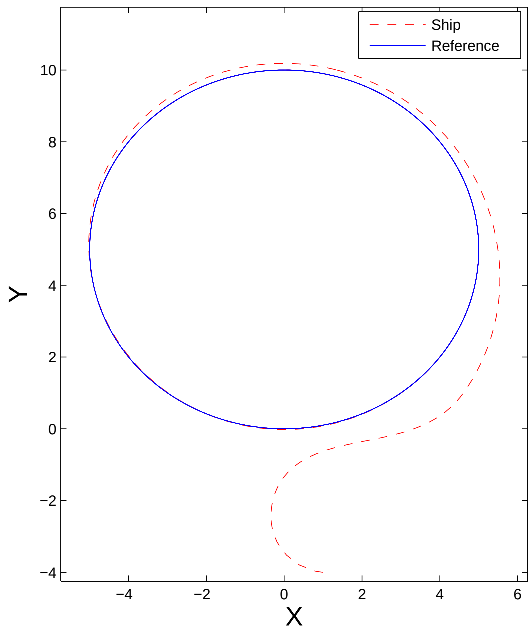

Figure 1: Desired and actual trajectory for underactuated ship.

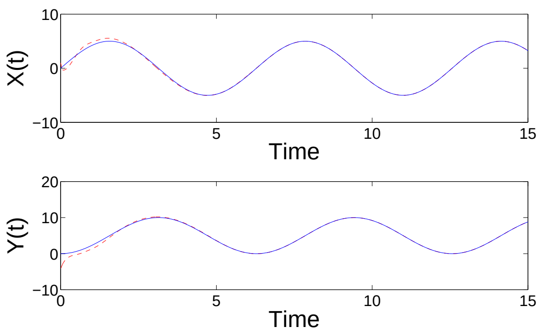

Figure 2: The time evolution of the position (- -) along with the respective desired trajectory (–).

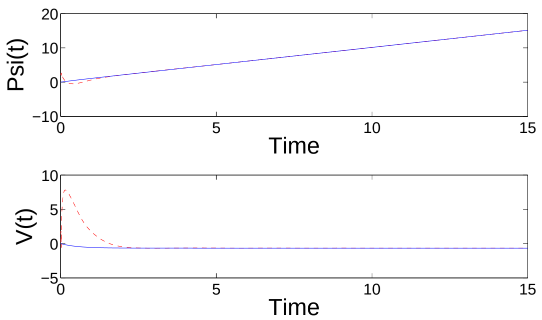

Figure 3: The time evolution of the orientation and sway velocity (- -) along with the respective desired trajectories (–).

5 Conclusions

In this paper, we have proposed a flatness-based approach to design a trajectory tracking controller for a simplified underactuated ship model. Precomputing the necessary off-line trajectory, as made in (Sira-Ramírez, 1999), the approach is based on showing a systematic trajectory design computing the flat outputs of the linearized system by the inverse of the controllability matrix associated with the Kronecker controllability indices.

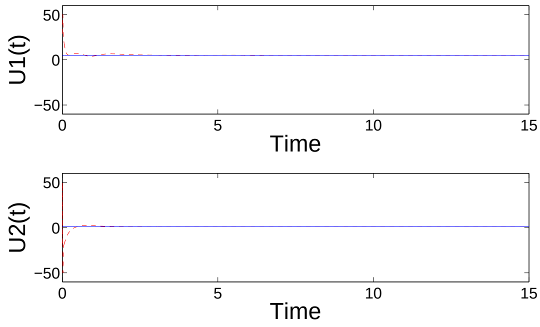

Figure 4: Control Inputs.

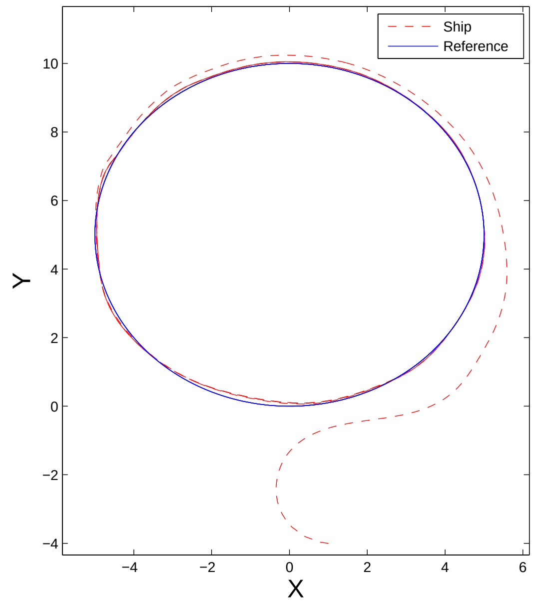

Figure 5: Circular path tracking performance under external sinusoidal perturbation.

The proposed feedback controller uses the off-line computed open loop control signals complemented with a linearization based feedback controller, providing the necessary on-line correction to track the desired trajectory.

The first simulation illustrates the tracking errors converging to zero exponentially, thereby ensuring that the ship perfectly follows the trajectory specified previously.

In the presence of external sinusoidal perturbations, it was observed that the system converges to a satisfactory approximation of desired path, but takes a little bit longer time than the first simulation. It is interesting to emphasize that the amplitude of the disturbance had 50% of the radius of the circular path desired. Both simulation results confirm the successful tracking performance and effectiveness of the proposed controller.

In further studies, we hope to validate the controller in a real platform. Moreover, the ideas shown in this paper can be applied to design controllers of other underactuated systems with similar model structures. There is an ongoing project which aims to extend this study to a more complex ship model where it adds the dynamic of the surge and yaw velocities.

Acknowledgment

The authors would like to to acknowledge the Brazilian institutions: ANP, FINEP, MCT and Petrobras for supporting the present study and the program PRH-PB 223.

References

Ayadi, M. (2002). *Contributions à la commande des systèmes linéaires plats de dimension finie*, PhD thesis, Institut National Polytechnique de Toulouse.

Behal, A., Dawson, D., Dixon, W. and Fang, Y. (2002). Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics, *Automatic Control, IEEE Transactions on* **47**(3): 495–500.

Berge, S. P., Ohtsu, K. and Fossen, T. I. (1999). Nonlinear control of ships minimizing the position tracking errors, *Modeling, Identification and Control* **20**(3): 177–187.

Chelouah, A. (1997). Extensions of differential flat fields and liouvillian systems, *Decision and Control, 1997., Proceedings of the 36th IEEE Conference on*, Vol. 5, pp. 4268–4273 vol.5.

Fliess, M., Lévine, J., Martin, P. and Rouchon, P. (1992). Sur les systèmes non linéaires différentiellement plats, *C. R. Acad. Sciences* **315**: 619–624.

Godhavn, J. (1996). Nonlinear tracking of underactuated surface vessels, *Decision and Control, 1996., Proceedings of the 35th IEEE Conference on*, Vol. 1, pp. 975–980 vol.1.

Limaverde F., J. O. A. and Fortaleza, E. L. F. (2013). Motion planning and tracking for marine vessels, *Proc. ASME 2012 32th International Conf. Ocean, Offshore and Arctic Engineering (OMAE'13)*, Nantes, France.

Olfati-Saber, R. (2001). *Nonlinear control of underactuated mechanical systems with application to robotics and aerospace vehicles*, PhD thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology.

Pan, J. and Do, K. D. (2009). *Control of Ships and Underwater Vehicles: Design for Underactuated and Nonlinear Marine Systems*, Advances in Industrial Control, Springer, Crawley, Australia.

Pettersen, K. and Egeland, O. (1996). Exponential stabilization of an underactuated surface vessel, *Decision and Control, 1996., Proceedings of the 35th IEEE*, Vol. 1, IEEE, pp. 967–972.

Pettersen, K. Y. and Nijmeijer, H. (1999). Global Practical Stabilization and Tracking for an Underactuated Ship - A Combined Averaging and Backstepping Approach, *Modeling, Identification and Control* **20**(4): 189–200.

Poljak, S. (1990). *On Controllability Kronecker Indices of Structured Systems*, Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University.

Reyhanoglu, M. (1996). Control and stabilization of an underactuated surface vessel, *Decision and Control, 1996., Proceedings of the 35th IEEE*, Vol. 3, IEEE, pp. 2371–2376.

Silverman, L. M. and Meadows, H. E. (1967). Controllability and observability in time-variable linear systems, *Siam Journal On Control* **5**(1): 64–73.

Sira-Ramírez, H. (1999). On the control of the underactuated ship: a trajectory planning approach, *Decision and Control, 1999. Proceedings of the 38th IEEE Conference on*, Vol. 3, pp. 2192–2197 vol.3.

Sira-Ramírez, H. and Agrawal, S. K. (2004). *Differentially Flat Systems*, Control Engineering Series, Marcel Dekker.

Sira-Ramírez, H. and Ibanez, C. (2000). The control of the hovercraft system: a flatness based approach, *Control Applications, 2000. Proceedings of the 2000 IEEE International Conference on*, pp. 692–697.

Toussaint, G. J., Basar, T. and Bullo, F. (2000). Optimal tracking control techniques for nonlinear underactuated systems, *Proceedings of the 39 th IEEE Conference on Decision and Control*, IEEE, pp. 2078–2083.