SYNCHRONIZATION OF HYPERCHAOTIC FINANCE SYSTEMS IN THE PRESENCE OF UNKNOWNS

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Abstract— This paper presents an adaptive algorithm to synchronize hyperchaotic finance systems in the presence of unknown system parameter and bounded disturbances. Based on Lyapunov-like analysis, an adaptive scheme is proposed to make the synchronization error asymptotically null. Simulation results are provided to demonstrate the effectiveness and feasibility of the proposed synchronization method.

Keywords— Synchronization, Lyapunov methods, chaotic finance systems.

Resumo— Neste artigo apresenta-se um algoritmo adaptativo para sincronizar sistemas financeiros hiper-caóticos na presença de parâmetros desconhecidos e distúrbios limitados. Com base em uma análise tipo *Lyapunov-like* um esquema adaptativo é proposto e provado que o erro de sincronização converge assintoticamente para zero. Simulações são apresentadas para mostrar a aplicação e desempenho do método de sincronização proposto.

Palavras-chave— Sincronização, métodos de Lyapunov, sistemas caóticos financeiros.

1 Introduction

Since the chaotic behavior in economics was discovered by Grandmont and Malgrange, 1986, a substantial amount of research has sought to link the role of chaos with the inherent randomness in macroeconomics models. This uncertainty can make precise financial forecasting very limited, compromising the effectiveness of financial policies adopted by the government to interfere in an economic system (Baumol and Quandt, 1985). In fact, the U.S. subprime mortgage crisis in 2007 illustrated how ineffective the government policies were to counteract the critical economic behavior that resulted in the global economic crisis, which may have resulted from the eminently chaotic nature of this financial system (Rui-Hong, 2009). Motivated by this theoretical and practical background, modeling and synchronization of nonlinear chaotic finance systems has been an active topic of study of many researchers (Jun-Hai and Yu-Shu, 2001; Zhao et al., 2011; Yu et al. 2012).

On the other hand, in recent years, chaos synchronization has been applied in several areas such as in electrical engineering (Xiao, 2009; Iu and Tse, 2000), biological systems (Arecchi et al., 2003), chemical engineering (Li et. al., 2003) and in secure communications (Lu et al, 2002; Sun et al., 2008; Dimassi and Loría, 2011).

Several methods have been proposed to achieve synchronization. For instance, nonlinear active control (Wang et al., 2009a, 2009b; Wang, 2010; Chen et al., 2009; Yassen, 2008), adaptive synchronization (Al-Sawada et al, 2010; Yang, 2011; Li, 2010; Ye and Deng, 2012), backstepping design (Njah and Sunday, 2009; Peng and Chen, 2008), and sliding mode control (Liu et al., 2009; Chen et al., 2007).

However, in most of the above works, it is assumed that the slave and master systems are perfectly or structurally known, i.e., the presence of unknown disturbances is not considered. For instance, the techniques in Al-Sawada et al. (2010), Yang (2011), Li (2010), Ye and Deng (2012), Zhao et al. (2011), and Mahmoud (2012) achieve asymptotic convergence of the synchronization error to zero under the crucial assumption of the inexistence of disturbances. Although the assumption of free disturbance may be interesting from the theoretical point of view, from the practical perspective it is a restrictive assumption since the presence of disturbances are, in general, unavoidable. Typical disturbances include state-dependent and time-dependent functions, which can be introduced, for instance, by unexpected changes in the system dynamic due to faults, changes in operation conditions, aging of equipment, and so on.

Moreover, it is well-known that adaptive laws designed for the disturbance or modeling error free case may suffer from parameter drift. In fact, this lack of robustness in adaptive systems in the presence of unmodeled dynamics or bounded disturbances was reported in the early 1980s. Several robust modifications to counteract this have been proposed since then (Ioannou and Sun, 1995).

Motivated by the previous facts, in this paper we propose a robust adaptive synchronization method to control the hyperchaotic finance system described in Yu et al. (2012), when bounded disturbances are present. In addition, it is assumed that the initial conditions and parameters of the finance system are unknown. Based on Lyapunov-like analysis (Slotine and Li, 1991), the proposed controller ensures the convergence of the synchronization error to zero, even in the presence of the aforementioned uncertainties.

2 Problem Formulation

Consider the hyperchaotic finance system described by

$$\dot{x}_s = f_s(x_s) + G_s(x_s)\theta + d_s(x_s,t) + u \tag{1}$$

where $x_s \in M \subset \Re^4$ is the state of the slave system, M is a compact set, $u \in \Re^4$ is the control input, $f_s(.)$ and $G_s(.)$ are known maps, $d_s(.)$ is an unknown disturbance and θ is an unknown parameter vector,

$$G_s(x_s) = \begin{bmatrix} x_{s1} & 0 & 0 & 0 & 0 \\ 0 & -x_{s2} & 0 & 0 & 0 \\ 0 & 0 & -x_{s3} & 0 & 0 \\ 0 & 0 & 0 & -x_{s1}x_{s2} & -x_{s4} \end{bmatrix} \tag{2}$$

$$f_s(x_s) = \begin{bmatrix} x_{s3} + x_{s4} + x_{s1}x_{s2} \\ 1 - x_{s1}^2 \\ -x_{s1} \\ 0 \end{bmatrix} \tag{3}$$

and

$$\theta = \begin{bmatrix} a \\ b \\ c \\ d \\ k \end{bmatrix} \tag{4}$$

where x_{s1} is the interest rate, x_{s2} is the investment demand, x_{s3} the price index, and x_{s4} is the average profit margin. Moreover, the parameter a denotes the savings, b denotes the investment cost, and c denotes the commodities demand elasticity. The parameters d and k are associated with the average profit margin.

Remark 1: In case that $d_s(x_s,t) = 0$ and $u = 0$, system (1)-(4) becomes the hyperchaotic finance system introduced in Yu et al., (2012).

We assume that the following can be established.

Assumption 1: On the region $\Re^4 \times [0,\infty)$

$$\|d_s(x,t)\| \le d_{s0} \tag{5}$$

where d_{s0} is a positive constant, such that $d_{s0} < \bar{d}_s$ and $\bar{d}_s$ is a known constant.

Assumption 2: The parameter $\|\theta\|$ is upper bounded by a known positive constant $\bar{\theta}$, such that $\bar{\theta} > \|\theta\|$.

Remark 2: Assumption 1 is quite natural since system (1) evolves in a compact set.

Remark 3: Notice that when a = 0.9, b = 0.2, c = 1.5, d = 0.2, and k = 0.17, system (1) shows hyperchaotic behavior (Yu et al., 2012).

In order to have a well-posed problem, without loss of generality, we consider the master system as

$$\dot{x}_m = f_m(x_m) + G_m(x_m)\theta \tag{6}$$

where $x_m \in \Re^4$, θ is a known parameter vector and

$$G_m(x_m) = \begin{bmatrix} x_{m1} & 0 & 0 & 0 & 0 \\ 0 & -x_{m2} & 0 & 0 & 0 \\ 0 & 0 & -x_{m3} & 0 & 0 \\ 0 & 0 & 0 & -x_{m1}x_{m2} & -x_{m4} \end{bmatrix} \tag{7}$$

and

$$f_m(x_m) = \begin{bmatrix} x_{m3} + x_{m4} + x_{m1}x_{m2} \\ 1 - x_{m1}^2 \\ -x_{m1} \\ 0 \end{bmatrix} \tag{8}$$

Hence, our aim is to design a feedback control u, such that the state x_s of the slave hyperchaotic system (1) tracks the state x_m of the master system (6).

Define the synchronization error $e(t) = x_s - x_m$. Then, from (1) and (6), we obtain the synchronization error equation

$$\dot{e} = f_s - f_m - (G_s - G_m)(\hat{\theta} - \tilde{\theta}) + d_s + u \tag{9}$$

Remark 4: It should be noted that in our formulation, for sake of simplicity, it was considered that $f_m(\cdot)$ and $f_s(\cdot)$ have similar structure. However, these nonlinear mappings can be unrelated, for instance, to include a priori knowledge of the disturbances.

3 Adaptive Synchronization

In this section, we considered the problem of asymptotic adaptive synchronization in the presence of unknown parameter and bounded disturbances. It is shown by using Lyapunov-like analysis that the synchronization error converges asymptotically to zero. The control law is motivated by Vargas and Hemerly, (2008).

Theorem 1: Consider the slave (1) and master (6) chaotic systems, which satisfy Assumptions 1-2, and the control law

$$u = -A(x_s - x_m) - (f_s - f_m) + (G_s - G_m)\hat{\theta} + u_r \quad (10)$$

with

$$u_r = -\frac{\gamma_3 e}{\lambda_{\min}(K)\left[\|e\| + \gamma_1 \exp(-\gamma_o t)\right]} \quad (11)$$

$$\dot{\hat{\theta}} = -\gamma_\theta\left[\gamma_2\|e\|\hat{\theta} + (G_s - G_m)^T Ke\right] \quad (12)$$

where

$$\begin{aligned} &PA + P^T A = Q,\ A > 0,\ P = P^T > 0, \\ &Q > 0 \quad K = P + P^T,\ \gamma_o \geq 0,\ \gamma_1 > 0, \\ &\gamma_2 > 0,\ \gamma_\theta > 0,\ \gamma_3 = 2\|K\|_F \bar{d}_0 + \gamma_2 \bar{\theta}^2, \\ &\gamma_4 = \gamma_{\min}(Q),\ \gamma_5 = \gamma_3(1+\gamma_1)/2,\ \text{and} \\ &\|K\|_F \text{ is the Frobenius norm of } K. \end{aligned} \quad (13)$$

Then, the slave and master systems synchronize, i.e., $\lim_{t\to\infty} e(t) = 0$.

Proof: Consider the Lyapunov function candidate

$$V = e^T Pe + \frac{1}{2}\tilde{\theta}^T \gamma_\theta^{-1}\tilde{\theta} \quad (14)$$

where $\tilde{\theta} = \hat{\theta} - \theta$.

The time derivative of (14) results

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} + \dot{\tilde{\theta}}^T \gamma_\theta^{-1}\tilde{\theta} \quad (15)$$

On the other hand, by using (10), the closed-loop synchronization error can be written as

$$\dot{e} = -Ae + (G_s - G_m)\tilde{\theta} + d + u_r \quad (16)$$

By evaluating (15) along the trajectories of (12) and (16), we obtain

$$\begin{aligned} \dot{V} &= e^T(P+P^T)(f_s - f_m) + e^T(P+P^T)(d+u) \\ &\quad + e^T(P+P^T)(G_s - G_m)(\hat{\theta} - \tilde{\theta}) \\ &+ \tilde{\theta}^T\gamma_\theta^{-1}\{-\gamma_\theta[\gamma_2\|e\|\hat{\theta} + (G_s - G_m)^T ke]\} \end{aligned} \quad (17)$$

Rewriting (17), results

$$\begin{aligned} \dot{V} &= e^T K \\ &\quad \cdot\left[f_s - f_m + d + u + (G_s - G_m)\hat{\theta} - (G_s - G_m)\tilde{\theta}\right] \\ &\quad - \tilde{\theta}^T\gamma_2\|e\|\hat{\theta} - \tilde{\theta}^T(G_s - G_m)^T Ke \end{aligned} \quad (18)$$

which, by using (10) , becomes

$$\dot{V} = -e^T Qe + e^T Ku_r + e^T Kd - \tilde{\theta}^T\gamma_2\|e\|\hat{\theta} \quad (19)$$

Since $\tilde{\theta} = \hat{\theta} - \theta$, it can be established that

$$\tilde{\theta}\hat{\theta} = \frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\hat{\theta}^2 - \frac{1}{2}\theta^2 \quad (20)$$

Thus by employing (11) and (20), (19) implies

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min}(Q)\|e\|^2 - \frac{\gamma_3\|e\|^2}{\left[\|e\| + \gamma_1\exp(-\gamma_0 t)\right]} \\ &\quad + \bar{d}_0\|K\|_F\|e\| + \frac{\gamma_2\|\theta\|^2\|e\|}{2} - \frac{\gamma_2}{2}\|\tilde{\theta}\|^2\|e\| \end{aligned} \quad (21)$$

By using definition (13), (21) can be written as

$$\begin{aligned} \dot{V} &\leq -\gamma_4\|e\|^2 - \frac{\gamma_3\|e\|^2 - \frac{\gamma_3}{2}\left(\|e\| + \gamma_1\exp(-\gamma_o t)\right)\|e\|}{\|e\| + \gamma_1\exp(-\gamma_o t)} \\ &\quad - \frac{\gamma_2}{2}\|\tilde{\theta}\|^2\|e\| \\ &\leq -\|e\|\left(\gamma_4\|e\| + \frac{\gamma_2}{2}\|\tilde{\theta}\|^2 - \gamma_5\right) \end{aligned} \quad (22)$$

Hence, $\dot{V} \leq 0$ as long as

$$\|e\| \geq \frac{\gamma_5}{\gamma 4} \equiv \alpha_e$$

or

$$\|\tilde{\theta}\| \geq \left(\frac{2\gamma_5}{\gamma_2}\right)^{1/2} \equiv \alpha_{\tilde{\theta}}$$

Thus, since α_e and $\alpha_{\tilde{\theta}}$ are constants, by employing usual Lyapunov arguments (Slotine and Li, 1991), we concluded that $e(t)$ and $\tilde{\theta}(t)$ are uniformly bounded.

Additionally, the first inequality in (22) can be rewritten as

$$\dot{V} \leq -\gamma_4\|e\|^2 - \frac{\gamma_3\|e\|\left[\|e\| - \gamma_1\exp(-\gamma_o t)\right]/2}{\|e\| + \gamma_1\exp(-\gamma_o t)} \quad (23)$$

To show that the synchronization error converges asymptotically to zero, define a region Ω in the synchronization error space by

$$\Omega = \{ e \mid \|e\| \le \gamma_1 \exp(-\gamma_0 t) \} \tag{24}$$

Then, in case $\|e\| > \gamma_1 \exp(-\gamma_0 t)$ (or $e \in \Omega^c$), we have

$$\dot{V} \le -\gamma_4 \|e\|^2 \tag{25}$$

Further, since V is bounded from below and non-increasing with time, we have

$$\lim_{t\to\infty} \int_0^t \|e(\tau)\|^2 d\tau \le \frac{V(0) - V_\infty}{\gamma_4} < \infty \tag{26}$$

where $\lim_{t\to\infty} V(t) = V_\infty < \infty$. Notice that, based on (16), with the bounds on $e, \tilde{\theta}, d$ and u_r, $\dot{e}$ is also bounded. Thus, $\dot{V}$ is uniformly continuous. Hence, by applying the Barbalat's Lemma (Slotine and Li, 1991), we conclude that $\lim_{t\to\infty} e(t) = 0$ for all $e \in \Omega^c$.

Once the synchronization error $e(t)$ has entered Ω, it will remain in Ω forever, due to (24) and (25). Consequently, we conclude that $\lim_{t\to\infty} e(t) = 0$ holds in the large, i.e., whatever the initial value of $e(t)$ (inside or outside Ω).

4 Simulations

In this section, we illustrate the application of the proposed synchronization method. Disturbances with practical meaning were considered in the simulations.

It was considered that $x_m = [1 \quad 2 \quad 0.5 \quad 0.5]$ and $x_s = [-1 \quad 6 \quad 4 \quad -2]$. Hence, to obtain the synchronization of the slave system (1) and the master system (6), the control laws (10)-(11) and adaptation mechanism (12) were employed.

The parameters used in the simulations for the hyperchaotic system were a = 0.9, b = 0.2, c = 1.5, d = 0.2 and k = 0.017. The others design parameters were chosen as $k = 1$, $\hat{\theta}(0) = 0$, $\lambda_0 = 0.01$, $\gamma_1 = 1$, $\gamma_2 = 0.001$ $\gamma_\theta = 0.05$, and $P = diag(0.0001, 0.1, 0.05, 0.01)$.

We consider the presence of state/time-dependent disturbances of the form

$$d_s(x_s, t) = \begin{bmatrix} 8\sin(7t)x_{s2}(t) \\ 7\cos(9t)x_{s2}(t) \\ (5\sin(t) + 6\cos(3t))x_{s2}(t) \\ 10\sin(3t)x_{s2}(t) \end{bmatrix} \tag{27}$$

The above disturbance can be interpreted as unmodeled dynamics, which can be related with fluctuations in the market caused by speculative behavior. It should be noted that these disturbances can often be dependent of the time and investment demand, in practice.

Figures 1-9 show the performances obtained with the proposed scheme. From Figures 19-22, it should be highlighted the fast synchronization between the master and slave systems. It means that the proposed adaptive controller is robust and can achieve quick synchronization, even in the presence of several classes of disturbances.

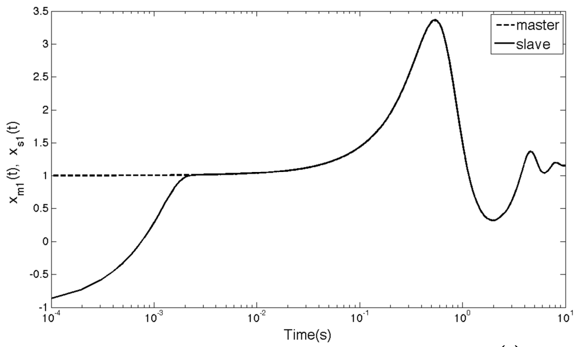

Figure 1. Performance for the synchronization of $x_{s1}(t)$.

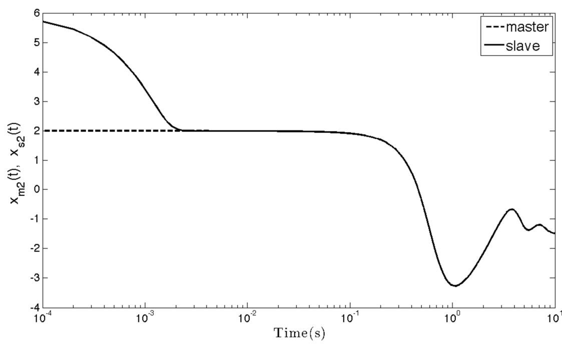

Figure 2. Performance for the synchronization of $x_{s2}(t)$.

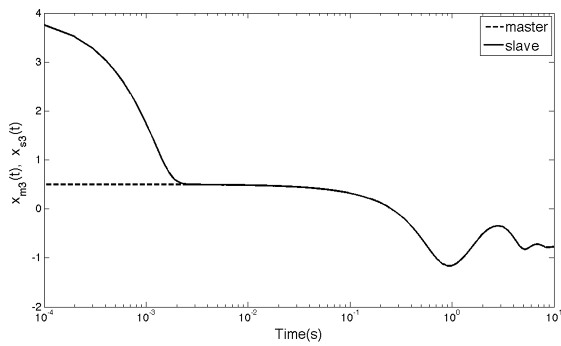

Figure 3. Performance for the synchronization of $x_{s3}(t)$.

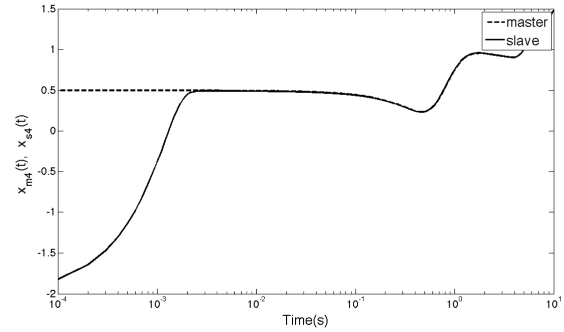

Figure 4. Performance for the synchronization of $x_{s4}(t)$.

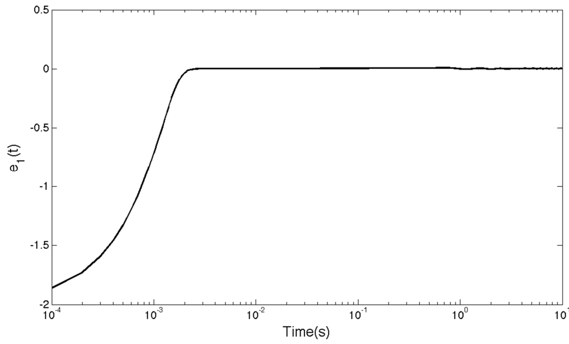

Figure 5. Synchronization error $(x_{s1}(t) - x_{m1}(t))$.

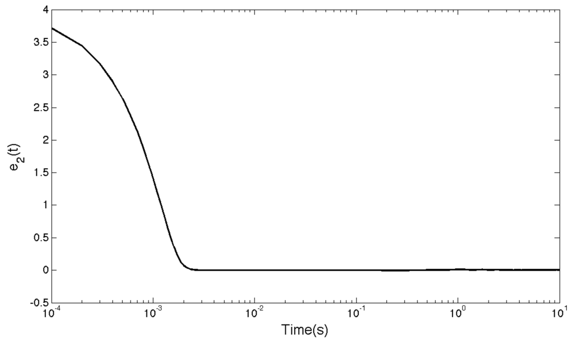

Figure 6. Synchronization error $(x_{s2}(t) - x_{m2}(t))$.

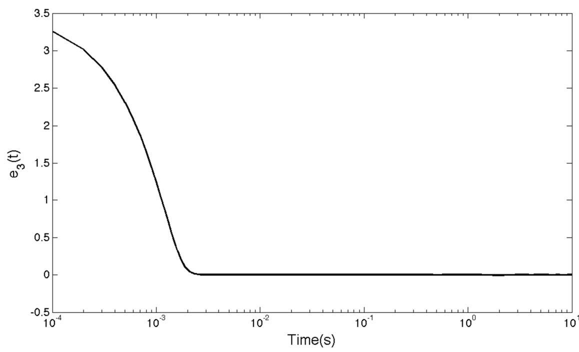

Figure 7. Synchronization error $(x_{s3}(t) - x_{m3}(t))$.

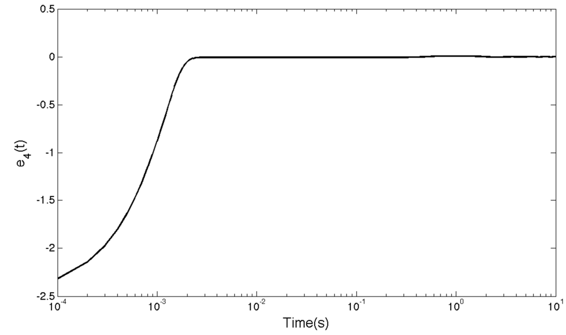

Figure 8. Synchronization error $(x_{s4}(t) - x_{m4}(t))$.

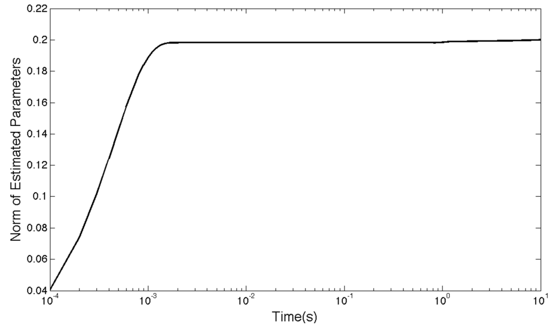

Figure 9. Norm of the estimated parameters $\|\hat{\theta}\|$.

5 Conclusions

In this paper, we have proposed an adaptive controller for synchronization of hyperchaotic finance systems, which can be affected by uncertainties, such as unknown system parameters and time-dependent or/and state-dependent disturbances. Based on Lyapunov-like analysis, an adaptive control system was proposed to ensure the asymptotic convergence of the residual synchronization error to zero, even in the presence of the aforementioned uncertainties. Simulation results were presented to illustrate the theoretical results and the application of the proposed scheme.

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