TRAJECTORY TRACKING FOR A FOUR ROTOR MINI-QUADROTOR

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Abstract— This paper presents the design of a controller that allows a four-rotor helicopter to track a desired trajectory in the 3D space. To this aim, a dynamic model obtained from Euler-Lagrange equations is used to describe the robot. Such model is represented by numerical methods and, from this approach, the control actions for the operation of the system are obtained. The proposed controller is easy implementation and presents a good performance.

Keywords— Trajectory Tracking; Quadrotor; Numerical Methods; Linear Algebra.

Resumo— Este artigo apresenta o projeto de um controlador que permite a um helicóptero de quatro rotores rastrear uma trajetória desejada no espaço 3D. Para este objetivo, um modelo dinâmico obtido a partir das equações de Euler-Lagrange é usado para descrever o robô. Esse modelo é representado por meio de métodos numéricos e, a partir desta abordagem, as acções de controle para a operação do sistema, são obtidos. O controlador proposto é simples e apresenta um bom desempenho.

Keywords— Rastreamento de trajetória; Quadrimotor; Métodos Numéricos; Álgebra Linear.

1 Introduction

In the last decades, the research effort related to Unmanned Aerial Vehicles (UAV) has grown substantially, aiming at either military or civil applications, such as inspection of large areas in public safety applications, natural risk management, inspection services of power lines, intervention in hostile environments, infrastructure maintenance and precision agriculture. In such cases, the use of a UAV is extremely advantageous, compared to the use of one or even several Unmanned Ground Vehicles (UGV), due to its 3D mobility.

Unmanned aerial vehicles can be classified as fixed-wing, rotary-wing and blimps. The main advantage of rotary-wing over fixed-wing aircraft is the ability of hovering and having omnidirectional movement. A disadvantage is, however, a relatively higher power consumption during the flight. Among the rotary-wing aircraft classification, a quadrotor is much simpler and easier to build in comparison to a classical helicopter, since it has no swashplate and is controlled by varying only the angular velocity of each of the four motors.

The control of quadrotor helicopters has attracted the attention of many researchers in the past few years. In the literature, different control strategies have been proposed, some of them use linear control techniques. In (How et al., 2008), the LQR was used for accurate orientation and position control oft MIT's RAVEN quadrotors. An approach based on switched dynamics and gain scheduling was proposed in (Gillula et al., 2011), that allowed a small quadrotor to perform acrobatic maneuvers. In this method, the behavior of the systems is approximated as a discrete set of simpler hybrid modes representing the dynamics in specific portions of the state space.

In (Kendoul et al., 2010), the design of a nonlinear flight control system and its implementation on a quadrotor are presented. The controller was designed by deriving a mathematical model of the quadrotor dynamics and exploiting its structural properties to transform it into two cascaded subsystems (attitude and translation) coupled by nonlinear interconnection term. Nonlinear controller based on the dynamic inversion technique can also by used for acrobatic and aggressive maneuvers control, as demonstrated in (Mellinger and Kumar, 2011). The developed algorithm generates optimal trajectories through a sequence of 3D positions and yaw angles.

In this paper it is presented a control technique which is able to follow piecewise continuous trajectories with piecewise continuous derivatives. It represents a new control approach whose originality is based on the application of numerical methods and linear algebra for trajectory tracking of a quadrotor. This simple approach suggests that knowing the value of the desired state, you can find a value for the control action, which forces the system to move from its current state to the desired one. In (Scaglia et al., 2009) and

(Rosales et al., 2011) it is introduced the numerical method-based controller, where the control law depends on the chosen numerical approximation. This controller allows trajectory tracking control as well as positioning control, because it only depends on the reference values. The paper is organized as follows: in section 2, the dynamic model of a four rotors helicopter is presented. In section 3, the helicopter model is aproximated by using numerical methods and the expression of the proposed controller is obtained. In section 4, simulation results of the control algorithm validate the theoretical results and show the characteristics of the proposed controller, and finally in section 5 experimental results with the control system are showed. In section 6, the conclusions of the paper and the proposals for future work, are presented.

2 Dynamic Model of a Quadrotor

In this section, the dynamic model of the four-rotor helicopter using Euler-Lagrange equations is obtained. This can be described in full in (Castillo et al., 2005). The generalized coordinates of the aerial vehicle are

$$q = (x, y, z, \phi, \theta, \psi) \ \in \Re^6 \tag{1}$$

where $\xi = (x, y, z)$ denotes the position of center of mass of the helicopter related to the inertial frame $\langle e \rangle$, and $\eta = (\phi, \theta, \psi) \in \ \Re^3$ are the Euler angles (with ϕ being the roll angle, θ the pitch angle and ψ the yaw angle in the spatial frame $\langle s \rangle$). They represent the helicopter orientation as shown in Figure 1.

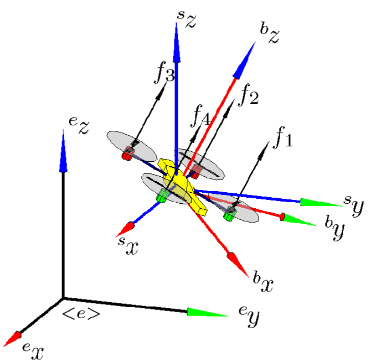


Figure 1: Diagram of a 6-DOF quadrotor and the associated frames e_i, s_i and b_i represent the inertial, spatial and body frame, respectively.

Defining the Lagrangian:

$$L_{(q,\dot{q})} = T_{trans} + T_{rot} - U \tag{2}$$

where $T_{trans} = \frac{m}{2}\dot{\xi}^T\dot{\xi}$ is the translational kinetic energy, $T_{rot} = \frac{1}{2}\omega^T I \omega$ is the rotational kinetic energy, $U = mgz$ is the potential energy of the system, z is the vehicle height, m denotes the helicopter mass, ω is the angular velocity, I is the inertia matrix and g is the gravitational acceleration. The full dynamic model of the helicopter is obtained from the Euler-Lagrange equations with external generalized forces:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} F_\xi \\ \tau \end{bmatrix} \tag{3}$$

where $F_\xi = RF \ \in \ \Re^3$ is the translational force applied to the vehicle caused by the main control input and R is a rotational matrix $R(\phi, \theta, \psi)$ that represent the orientation of the quadrotor relative to inertial frame and $\tau \in \ \Re^3$ represent the moment of pitch, roll and yaw. The principal input of the aircraft is the impulse of the four propellants,

$$F = [0 \ \ 0 \ \ u]^T \tag{4}$$

from (Kondak et al., 2007),

$$u = \sum_{i=1}^{4} f_i; \text{ with } f_i = k_i \omega_{mi}^2, \text{for } i = 1, ..., 4. \tag{5}$$

The parameter k_i is taken as a positive constant value and ω_i is the angular velocity of the $i - th$ motor. The generalized torques are:

$$\tau = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} (f_3 - f_1)l \\ (f_2 - f_4)l \\ \sum_{i=0}^{4} \tau_{Mi} \end{bmatrix} \tag{6}$$

where l is the distance between the motors and the center of gravity, and τ_{Mi} is the torque produced by the motor M_i, $i = 1, ..., 4$ around the center of gravity of the vehicle. Observing that the Lagrangian function does not contain terms in the kinetic energy combining $\dot{\xi}$ with $\dot{\eta}$, the Euler-Lagrange equations can be divided in the dynamics for the coordinates of ξ and the coordinates of η, as follow

$$m\ddot{\xi} + [0 \ \ 0 \ \ mg]^T = F_\xi \tag{7}$$

$$J\ddot{\eta} + \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}(\dot{\eta}^T J \dot{\eta}) = \tau \tag{8}$$

Defining the Coriolis terms as

$$C_{(\eta,\dot{\eta})}\dot{\eta} = \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}(\dot{\eta}^T J \dot{\eta}) \tag{9}$$

which contain the gyroscope and centrifugal effects associated with η. Finally we get,

$$\begin{aligned} m\ddot{\xi} + [0 \ \ 0 \ \ mg]^T &= F_\xi \\ J\ddot{\eta} + C_{(\eta,\dot{\eta})}\dot{\eta} &= \tau \end{aligned} \tag{10}$$

In order to simplify the model, the following change in the input variable is proposed

$$\tau = C_{(\eta,\dot{\eta})}\dot{\eta} + J\tilde{\tau} \tag{11}$$

where $\tilde{\tau} = [\tilde{\tau_\phi}, \tilde{\tau_\theta}, \tilde{\tau_\psi}]$, is the new input vector. Then

$$\ddot{\eta} = \tilde{\tau} \tag{12}$$

Finally, we obtain:

$$\begin{aligned} m\ddot{x} &= -u \sin\theta \\ m\ddot{y} &= u\cos\theta \sin\phi \\ m\ddot{z} &= u\cos\theta\cos\phi - mg \\ \ddot{\phi} &= \tilde{\tau}_\phi \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\psi} &= \tilde{\tau}_\psi \end{aligned} \tag{13}$$

where x and y are the coordinates in the horizontal plane, z is the vertical position, and $\tilde{\tau_\phi}$, $\tilde{\tau_\theta}$ and $\tilde{\tau_\psi}$ are the roll, pitch and yaw torques respectively, which are related to the generalized torques τ_ϕ, τ_θ and τ_ψ by Eq. (11).

3 Controller Design

3.1 Problem Statement

Consider the following differential equation,

$$\dot{y} = f_{(y,t)} \quad y_{(0)} = y_0 \tag{14}$$

With the objective to determine the value of $y_{(t)}$ in discrete time instants $t = nTo$, where To is the sample period and $n \,\epsilon\, \{0, 1, 2, 3, ...\}$. The value of the variable $y_{(t)}$ for $t = nTo$, will be symbolized as $y_{(n)}$. Thus, if you want to calculate $y_{(n+1)}$ knowing the value of $y_{(n)}$, (14) should be integrated in the interval $nTo \leq t \leq (n+1)To$, as

$$y_{(n+1)} = y_{(n)} + \int_{nTo}^{(n+1)To} f_{(y,t)} dt \tag{15}$$

An approximate value of $y_{(n+1)}$ can be obtained using numerical methods to calculate the integral on the second member of (15). For example, it can be calculated as,

$$y_{(n+1)} \cong y_{(n)} + To f_{(y_{(n)}, t_{(n)})} \tag{16}$$

which is called the Euler approximation. Even existing other numerical methods to approximate the integral in equation (15), in this paper we apply the Euler approximation to get the discrete dynamic model of a quadrotor. Then, based on this model, it will be obtained the optimal control actions that allow the helicopter to follow a path previously established, by ultimately solving a mean squares algebraic problem.

3.2 Controller design

In this section, it is designed a control law capable of generating the signals $[u, \tilde{\tau_\phi}, \tilde{\tau_\theta}, \tilde{\tau_\psi}]$, with the objective that the helicopter position $[X_{(t)}, Y_{(t)}, Z_{(t)}, \Psi_{(t)}]$ follows the desired trajectory $[Xd_{(t)}, Yd_{(t)}, Zd_{(t)}, \Psi d_{(t)}]$. The relationship between the generalized pairs and the new inputs $(\tilde{\tau})$ is given by (11).
The first step in the controller design involves expressing the model given in equation (13) in state form as a set of linear first order differential equations,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \\ \dot{x_7} \\ \dot{x_8} \\ \dot{x_9} \\ \dot{x_{10}} \\ \dot{x_{11}} \\ \dot{x_{12}} \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{u}{m}\sin x_9 \\ x_4 \\ \frac{u}{m}\cos x_9 \sin x_7 \\ x_6 \\ \frac{u}{m}\cos x_9 \cos x_7 - g \\ x_8 \\ \tilde{\tau_\phi} \\ x_{10} \\ \tilde{\tau_\theta} \\ x_{12} \\ \tilde{\tau_\psi} \end{bmatrix} \text{ where } \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} \tag{17}$$

Using the Euler approximation and expressing in matrix form and operating, we have

$$\underbrace{\begin{bmatrix} a_{11}To & 0 & 0 & 0 \\ a_{21}To & 0 & 0 & 0 \\ a_{31}To & 0 & 0 & 0 \\ 0 & To & 0 & 0 \\ 0 & 0 & To & 0 \\ 0 & 0 & 0 & To \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \frac{u_{(n)}}{m} \\ \tilde{\tau}_{\phi(n)} \\ \tilde{\tau}_{\theta(n)} \\ \tilde{\tau}_{\psi(n)} \end{bmatrix}}_{w} = \underbrace{\begin{bmatrix} \Delta x_{2(n)} \\ \Delta x_{4(n)} \\ \Delta x_{6(n)} + gTo \\ \Delta x_{8(n)} \\ \Delta x_{10(n)} \\ \Delta x_{12(n)} \\ \Delta x_{1(n)} - Tox_{2(n)} \\ \Delta x_{3(n)} - Tox_{4(n)} \\ \Delta x_{5(n)} - Tox_{6(n)} \\ \Delta x_{7(n)} - Tox_{8(n)} \\ \Delta x_{9(n)} - Tox_{10(n)} \\ \Delta x_{11(n)} - Tox_{12(n)} \end{bmatrix}}_{b} \tag{18}$$

Where $a_{11} = -\sin x_{9(n)}$, $a_{21} = \cos x_{9(n)} \sin x_{7(n)}$ and $a_{31} = cx_{9(n)}cx_{7(n)}$ and was replacing the next relationship $\Delta x_{i(n)} = x_{i(n+1)} - x_{i(n)}$. This equation can be expressed in compact form as.

$$A\mathbf{w} = b \tag{19}$$

If the desired trajectory is given, $[Xd_{(n+1)}, Yd_{(n+1)}, Zd_{(n+1)}, \Psi d_{(n+1)}]^T$, then it can be taken into account to calculate the required control action $[u, \tilde{\tau_\phi}, \tilde{\tau_\theta}, \tilde{\tau_\psi}]^T$, that allows the helicopter to evolve from the present position to the desired trajectory.
Equation (18) represents a system of linear equations which allows at each sampling instant to calculate the control actions ($\mathbf{w}$) in order that the quadrotor achieves the desired trajectory. Now, it is necessary to specify the conditions for this system to have an exact solution.

The first condition for the system of (18) to have exact solution is that the first 6 equations and 4 unknown variables has exact solution. It can be concluded that these conditions are given by (21) and (22).

$$\begin{bmatrix} -\frac{\sin x9_{(n)}}{m} \\ \frac{\cos x9_{(n)} \sin x7_{(n)}}{m} \\ \frac{\cos x9_{(n)} \cos x7_{(n)}}{m} \end{bmatrix} \begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} \frac{\Delta x_2}{To} \\ \frac{\Delta x_4}{To} \\ \frac{\Delta x_6 + gTo}{To} \end{bmatrix} \quad (20)$$

$$\tan x7_{ez} = \frac{\Delta x_4}{\Delta x_6 + gTo} \quad (21)$$

$$\tan x9_{ez} = -\frac{\Delta x_2}{\Delta x_4} \sin x_{7ez} \quad (22)$$

From (21) and (22) the variable references $x7_{ez}$ and $x9_{ez}$ are obtained so that the system of equations (18) has exact solution and thus, the quadrotor can follow the reference trajectory. These variables represent the necessary orientations to allow the tracking error to tend to zero.

Additionally, from (18) it can be seen that in order that the system of equations have an exact solution, the rows of b corresponding to the zero rows of A must be equal to zero, then

$$x_{k(n)} = \frac{x_{jref(n+1)} - x_{j(n)}}{To} \quad (23)$$
$$j = \{1, 3, 5, 7, 9, 11\}$$
$$k = \{1, 3, 5, 7, 9, 11\}$$

From the previous equations, we obtain the speed references that make the quadrotor follow the desired trajectory.

In order that the tracking error tends to zero for all the state variable that represent the position and attitude (x_i for $i = 1, 3, 5, 7, 9, 11$) of the helicopter, the following expressions are defined,

$$x_{iref(n+1)} = xd_{i(n+1)} - k_{xi}(xd_{i(n)} - x_{i(n)}) \quad (24)$$
$$i = \{1, 3, 5, 7, 9, 11\}$$

where $0 < k_{x1}, k_{x3}, k_{x5}, k_{x7}, k_{x9}, k_{x11} < 1$.

After replacing (24) in (25), the velocities necessary for the tracking error to tend to zero are obtained. These values are the desired values to make it possible to follow the trajectory correctly and they are called with the subscript "d" to identify.

$$xd_{j(n+1)} = \frac{(x_{id(n+1)} - k_{xi}(x_{id(n)} - x_{i(n)})) - x_{i(n)}}{To} \quad (25)$$
$$i = \{1, 3, 5, 7, 9, 11\}$$
$$j = \{2, 4, 6, 8, 10, 12\}$$

We apply the same approach structure that the one expressed in (24) with the reference speed values obtained in (25) to make the speed quadrotor tend to the reference speed.

$$x_{jref(n+1)} = xd_{j(n+1)} - k_{xj}(xd_{j(n)} - x_{j(n)}) \quad (26)$$
$$j = \{2, 4, 6, 8, 10, 12\}$$

where $0 < k_{x2}, k_{x4}, k_{x6}, k_{x8}, k_{x10}, k_{x12} < 1$.

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{u_{(n)}}{m} \\ \tilde{\tau}_\phi \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\psi \end{bmatrix} = \frac{1}{To} \begin{bmatrix} \Delta x_2 \\ \Delta x_4 \\ \Delta x_6 + gTo \\ \Delta x_8 \\ \Delta x_{10} \\ \Delta x_{12} \end{bmatrix} \quad (27)$$

Equation (27) is solved using the pseudo-inverse matrix that represents the optimal least squares solution (Strang, 1980), and in this way, we obtain the control actions.

$$w = A^\dagger b \quad (28)$$

4 Simulation results

This subsection presents two flight simulations of the quadrotor in the 3D space, using the controller designed in Section 3. The goal of the simulations is to confirm the good performance of the control law. The first simulation shows the performance of the controller without model uncertainties and disturbance.

The simulation is performed using a simulator developed in the Matlab© platform, which considers an accurate model of the vehicle (Pizetta et al., 2012). The aircraft considerate in this work is an ArDrone Parrot©.

$$\begin{array}{lll} m = 0.380kg & l = 0.20m & T_o = \frac{1}{30}s \\ I = diag[9.57 & 18.57 & 25.55] \cdot 10^{-3} Nms^2/rad \end{array}$$

To check the performance of the proposed controller, a helical path was used as desired trajectory, centered at the origin of the inertial frame whit radius 2m and the quadrotor orientation (Ψ_d) set to $\pi/2$ radians. In order to demonstrate that the proposed controller can also be useful for position control, it is considered a reference point $(Xd, Yd, Zd, \Psi d) = (0m, 0m, 12m, -\pi/2rad)$, and a final motion to descend to the origin $(Xd, Yd, Zd, \Psi d) = (0m, 0m, 0m, 0rad)$ of the inertial frame to complete the simulation. The helical trajectory is generated with an upward velocity of $v_z = 0.8m/s$ and an angular velocity of $\omega = 1rad/s$. The initial position of the helicopter is at the frame origin and the trajectory starts at position $[0m; 0m; 2m]$. The selected control gains are given in Table 1.

Fig.2 shows the 3D representation of the position of the vehicle, which succeeds in reaching

Table 1: Control gains used in the first simulation.

$k_{x1} = 0.78$	$k_{x2} = 0.8$	$k_{x3} = 0.77$
$k_{x4} = 0.8$	$k_{x5} = 0.9$	$k_{x6} = 0.95$
$k_{x7} = 0.8$	$k_{x8} = 0.85$	$k_{x9} = 0.7$
$k_{x10} = 0.67$	$k_{x11} = 0.7$	$k_{x12} = 0.67$

and following the desired trajectory. Also it can be seen the evolution of the quadrotor to the desired set points during positioning control.

Fig. 3(a) shows the time evolution of coordinates ξ, which tend to the reference values. Figure 3(b) shows the time evolution of the attitude variables ϕ, θ and as ψ which also approximate their reference values and Fig. 3(c) represents the control errors which tend to zero, including those of the position control phase. Finally, Fig.3(d) represents the control actions $(u, \tau_\phi, \tau_\theta, \tau_\psi)$ expressed by (5) and (6), which are the real control actions directly applied to the vehicle.

5 Experimental Result

In this section is presented a real experiment of yaw control and altitude tracking. In spite of experiment displayed in this section is relatively simple, the objective is to show the stability and performance of the controller proposed in 3. In the experiment was used a quadrotor Parrot ArDrone©. The goal of the experiment is altitude tracking combined with yaw control. The altitude reference is giving by $z = 0.5 + 0.25\sin(\frac{\pi}{10}t)$ and a yaw reference of $\psi = \frac{-\pi}{4}$. In Fig.4(a) is possible to see the time evolution of the position variables, and it is proved that the helicopter track the altitude reference.

Fig.4(b) present the time evolution of the attitude variables is, and as from the start position ($\psi_0 \cong \frac{-\pi}{10}$) the quadrotor reach and maintain the reference value $\frac{-\pi}{4}$.

In 4(c) is showed the real action that was send to the ArDrone and is possible to observe that the last command that represent $\dot{Z}$, follow the variations of the altitude reference. Is possible to see the error in the x and y variables (drifting), and cause a undesirable variation in the evolution of the position variables. The cause of this problem is the data of the inertial sensors of the helicopter that affect the odometry. Is possible to resolve it throughout an extern position system, GPS o artificial vision system.

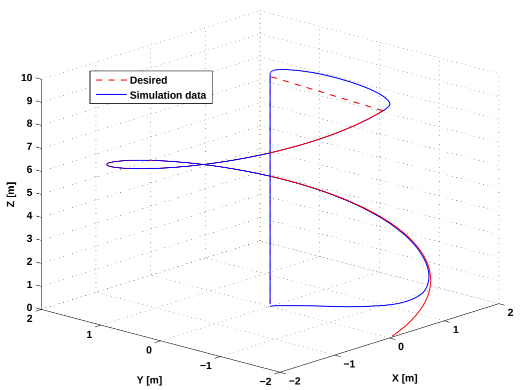
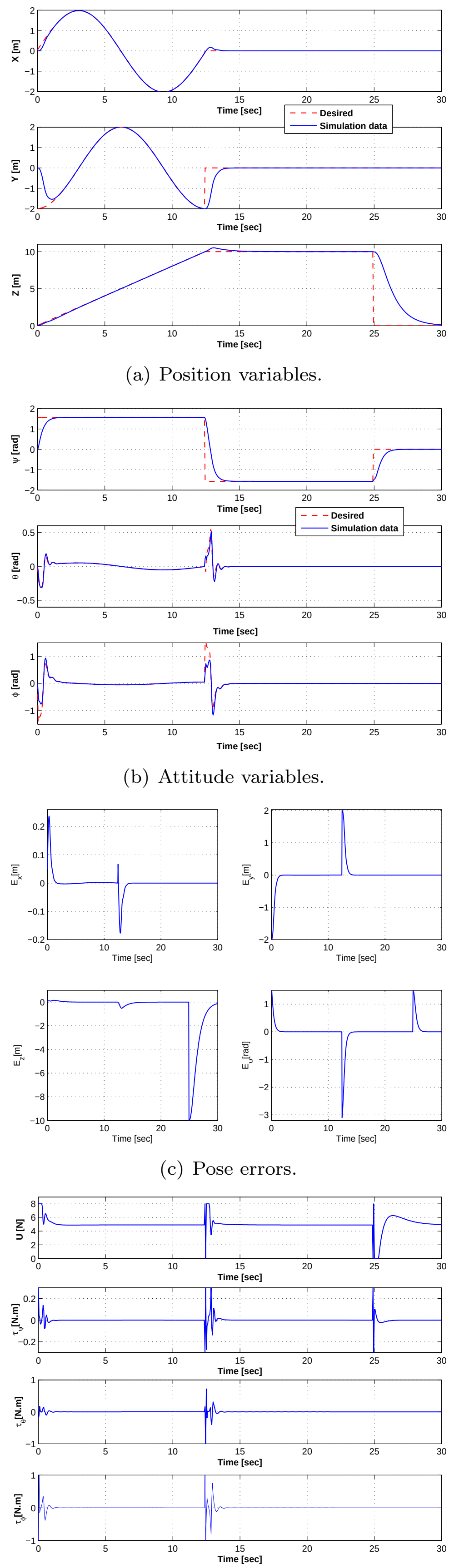


Figure 2: Evolution of the quadrotor position.



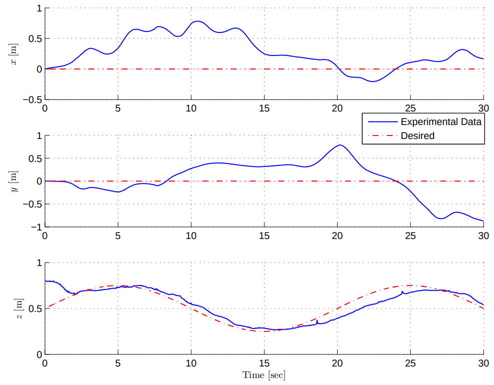
(a) Position variables.

(b) Attitude variables.

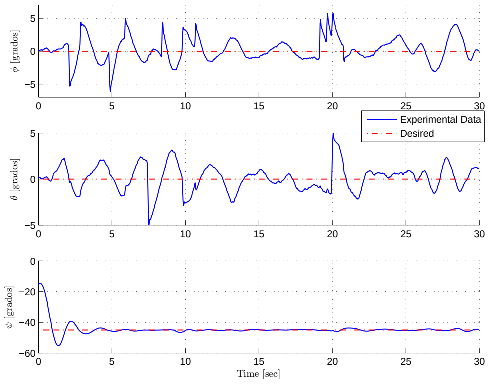
(c) Pose errors.

(d) Control actions.

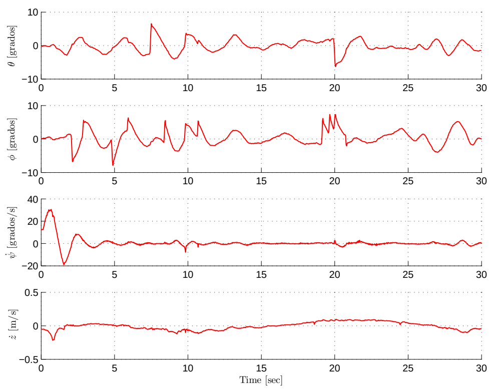
Figure 3: Time evolution during simulation of trajectory tracking of an helical reference.



(a) Position variables.



(b) Attitude variables.



(c) Control signals.

Figure 4: Time evolution during experimentation using an ArDrone Parrot quadrotor.

6 Conclusions

In this paper, it is presented the design of a trajectory controller for a four-rotors helicopter. To this aim, an approximation of the helicopter dynamic model using numerical methods is used. The proposed controller allows trajectory tracking as well as position control without switching the controller. In addition, the controller is easy to design and to implement with small computational complexity. It has been shown the good performance of the controller through simulations and real experiment, which shown the stability of the vehicle during the task execution.

As future work it can be mentioned the extension of the controller proposal to other unmanned aerial vehicles, and a systematic study of the robustness properties of the controller, as well as the analytical limitation of control actions to ensure its boundedness in any operation condition.

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