DYNAMIC MODEL OF A COMMERCIAL VEHICLE FOR STEERING CONTROL AND STATE ESTIMATION

Olmer García Bedoya †*, Janito Vaqueiro Ferreira*, Arthur de Miranda Neto*

*Autonomous Mobility Laboratory (LMA in Portuguese),FEM, UNICAMP
CEP 13083-860,Campinas, SP,Brasil

†Bolsista PEC/PG CAPES/CNPq-Brazil

Emails: olmerg@fem.unicamp.br, janito@fem.unicamp.br, arthur@fem.unicamp.br

Resumo— O projeto de um veículo robotizado do Laboratório de Mobilidade Autônoma (LMA) da UNICAMP se desenvolve numa plataforma veicular FIAT-PUNTO. Além de um conjunto de sensores, atuadores, mecanismos e componentes (hardware e/ou software), novas tecnologias devem ser desenvolvidas em prol da Automação, da Percepção, da Localização, da Navegação e do Controle. Este trabalho apresenta um modelo matemático não linear da dinâmica do movimento do carro e da dinâmica do sistema de direção. Para validar o modelo, simulações são realizadas sobre o projeto do controle da direção utilizando um Filtro de Kalman Estendido (EKF) para fazer a fusão de sensores que trabalham com diferentes taxas de amostragem, como o encoder óptico rotativo do motor da direção, o giroscópio e o sistema de posicionamento global (GPS)

Palavras-chave— Veículos autônomos, Sistemas realimentados ,Filtro de Kalman Estendido, modelo não linear

Abstract— The design of a robotic vehicle LMA at UNICAMP is developed over a model vehicle platform known as Fiat Punto. In addition to a set of sensors, actuators, mechanism and components (hardware and/or software), new technologies should be developed to support the Automation, the Control, the Perception, the Localization and the Navigation. This work presents a mathematical model of the nonlinear dynamics of the vehicle and the dynamic of the steering system. To validate the model, simulations are carried out over the design of the steering control system using an Extended Kalman Filter (EKF) for making a fusion of sensors that work at different sample times, such as the encoder of the steering motor, the gyroscope and the global position system (GPS).

Keywords— Autonomous vehicles, extended Kalman filter, feedback control, non linear model,steer-by-wire.

1 Introduction

Robotics is one of the main drivers for innovation. Especially looking at the development of applications related to defense system, it can provide a revolutionary level in terms of combat effectiveness. But the civil context is also one of the areas of great interest from Industry, with major growth and with large investment in the coming years. In this sense, especially for robot navigation, can be subdivided into two classes one on the environment previously known and the other on the environment unknown. The latter class is of great interest for the military industry.

In Brazil, the issue of land vehicles in outdoor environments is developed in universities and research centers, such as the VERO Project(De Paiva, 2010; Bueno, 2009), the CADU Project at UFMG[1], the UNIFEI autonomous vehicle at Itajubá/MG and the Carina Project at EESC-USP in São Carlos/SP. Companies begin to structure, focusing initially on teleoperated systems, but aiming that the evolution of these systems will become solutions to be applied in autonomous systems, which represent the technological objective to be achieved.

In mid-2008, interested in robotics and vehicular safety, the Autonomous Mobility Laboratory (LMA) was formed in the DMC-FEM at UNICAMP. The three main lines of study currently in LMA are: Perception, Navigation and Control. According (Thrun et al., 2006), these systems can be organized into five major functional groups: Interface Sensors, Perception, Control, Vehicle Interface and User Interface. According (Chen et al., 2008), this type of architecture facilitates the separation of functions and development, allowing for the definition of interfaces between the various subsystems.

Figura 1: Robotic platform VILMA (LMA's Intelligent Vehicle) based in Fiat Punto

This paper is part of the project to design and development the control and automation of the vehicle of LMA (figure 1)converting it from the traditional mechanical system to a drive-by-wire system, which include the automation of the direction, acceleration and braking system to be controlled from a software program. The control system to be developed will have the capacity of knowing when the user is trying to possess the

[1]http://www.youtube.com/watch?v=M4ZVRhNeKXU

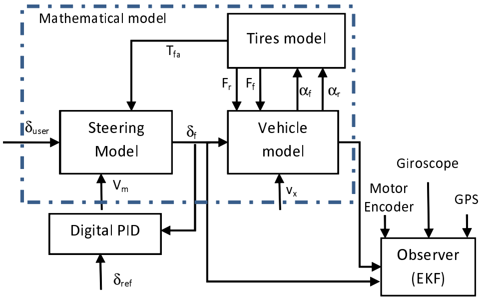

Figura 2: Block diagram of the simulation

control of the vehicle at the same time the system steer-by-wire is activated, so that the system can be switched from automatic to manual.

The paper can be resumed by the figure 2 and it is presented in four section. The first section presents the mathematical model, the second section introduces the observer and steering control design. The third section discuss the results of the simulations. Finally, the fourth section presents some conclusions and futures work.

2 Mathematical model

The mathematical model is divided in three modules (Figure 2), whose are based in the methodology used in (Avak, 2004) .The modules are: the mechanical dynamics of the vehicle, the tires model and the steering model. The model has three input: the steering torque of the user δ_{user}, the voltage input of the steering motor V_m and the longitudinal speed of the vehicle v_x.

2.1 Vehicle model

Three models has been studied about the vehicle. The first one is a kinematic model that assumes that does not exist slip or accelerations and that the kinematic of the car is the same as that of a bicycle(Snider, 2009). The second one the slip and the acceleration is considered but the model is also based in a single track model (Avak, 2004; Yih, 2005; Snider, 2009). The last one are based in a four-wheel vehicle model (FWVN)(Doumiati et al., 2010). In this work it is used the second model.

It is assumed that the vehicle body dynamics equations describe the behavior of the vehicle in translational and yaw motion (roll dynamics is neglected) and that both the two front wheels and the two rear wheels are lumped into one wheel at the center line of the vehicle (Figure 3).

The external forces acting over the vehicle, assuming v_x constant, are the lateral forces of the tires F_{fy} and F_{ry}. Applying the second law of Newton results the equation (1) .

$$m\mathbf{a_y} = cos\delta_f F_{fy} + f_{ry} \tag{1}$$

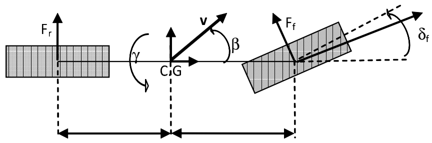

Figura 3: Single-track model

where, m is the mass of the car and $\mathbf{a_y}$ is the acceleration of the velocity vector plus the Coriolis acceleration caused by the rotation of the body.

$$\mathbf{a} = \gamma x \mathbf{v} + \dot{\mathbf{v}} = v_x\gamma + \dot{v}_y \tag{2}$$

where γ is the velocity angular of the vehicle. Taking equations (1) and (2) we obtain the first differential equation (3).

$$m(\dot{v_y} - \gamma v_x) = cos\delta_f F_{fy} + f_{ry} \tag{3}$$

The second differential equation (4) is based on Newton's second law for rotational motion regarding the the center of gravity (CG) of the vehicle.

$$J_v\dot{\gamma} = F_{fy}l_f cos\delta_f - F_{ry}l_r \tag{4}$$

where,J_v is the moment of inertia of the car, l_f is distance from the CG to the front wheel and l_r is distance from the CG to the rear wheel.

In the last equation v_x and v_y are in the local coordinate system, so transfering to global system without the non holonomic constraints (Pepy et al., 2006) result the following equations (5), (6) and (7).

$$\dot{x} = v_x cos\theta_c - v_y sin\theta_c \tag{5}$$

$$\dot{y} = v_x sin\theta_c + v_y cos\theta_c \tag{6}$$

$$\dot{\theta}_c = \gamma \tag{7}$$

Taking as state variables x, y , θ_c , v_y and γ and as inputs v_x, F_{ry}, F_{fy} and δ_f from (3), (4), (5), (6) and (7) we have the next state equation (8) which describe the movement of the CG of the car.

$$\mathbf{\dot{x}_c} = \begin{bmatrix} \dot{v_y} \\ \dot{\gamma} \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \frac{cos\delta_f F_{fy} + f_{ry}}{m} + \gamma \\ \frac{F_{fy}l_f cos\delta_f - F_{ry}l_r}{J_v} \\ v_x cos\theta - v_y sin\theta \\ v_x sin\theta + v_y cos\theta \\ \gamma \end{bmatrix} \tag{8}$$

Additionally, as it is presented in figure 2, two output variables of the car model are required to obtain the tires model. The front slip angle α_f, which is defined as the angle between the center line of the front wheel and the direction of the velocity of the front wheel, and the rear angle α_r, which is the angle between the center line of the

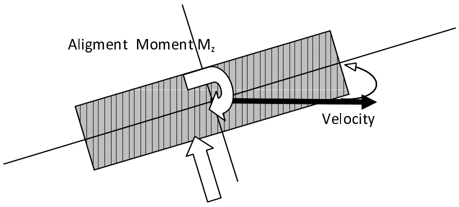

Figura 4: tire operating at slip angle

rear wheel and the direction of the velocity of the rear wheel. These variables can be described in terms of the state variables by the next set of equations (9).

$$\begin{aligned} \alpha_r &= atan\left(\frac{-v_y+l_r\gamma}{v_x}\right) \\ \alpha_f &= atan\left(\frac{-v_y+l_f\gamma}{v_x}\right) - \delta_f \end{aligned} \quad (9)$$

2.2 Tires model

Tires are perhaps the most important and difficult component of an automobile to model because in addition to supporting the vehicle and damping out road irregularities, the tires provide the longitudinal and lateral forces necessary to change the speed and direction of the vehicle. These forces are produced by the deformation of the tire where it contacts the road during acceleration, braking, and cornering.

Many models exist to represent this interaction, and the two most frequently approach used in the literature are the Pacejka's (Pacejka e Besselink, 1997) and the Dugoff's (Dugoff e Segel, 1970) formulas. Here in we use the Pacejka tire model where there is no particular physical basics for the structure of the equations chosen, and they fit a wide variety of tire constructions and operating conditions. For the lateral force and the aligning moment (figure 4) two variables are required, the slip angle, which is the angle between its direction of motion and the wheel plane, and the vertical force over the tire (F_{rz} and F_{fz}).

The graphics of the lateral force are presented in the figure 5 and the alignment moment in figure 6 was calculated using the formulas and values presented in (Yih e Gerdes, 2005) performed when $\mu = 1$, where μ is a coefficient that represents the adherence between the soil and the tires, which varies according to each type of soil.

2.3 Steering model

The steering system of the Fiat Punto is hydraulically assisted. Therefore, the model has three modules: the steering actuator, the mechanical system and the hydraulic system like the one presented in (Yih, 2005) with some modifications in the mechanical system. The relationship between the modules is described in figure 7. The input of the system are the torque of the driver (δ_{user}),

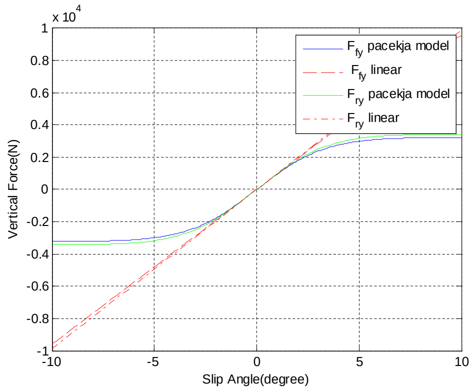

Figura 5: Lateral force of the front and rear tires in the bycicle model

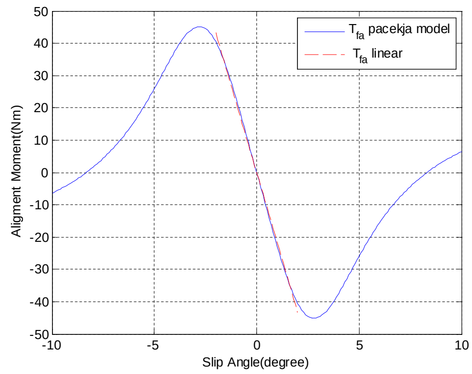

Figura 6: Aligment moment of the front and rear tires in the bycicle model

the pump flow rate (Q_s) which in this model will be ignored, the voltage of the motor supplied by PWM by the integrated servodriver (V_m) and the alignment moment of the front tire (T_{fa}).

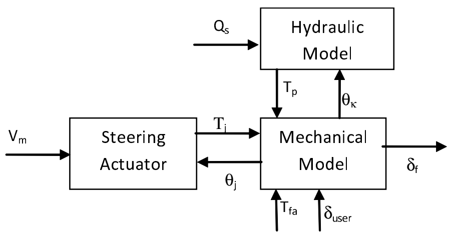

Figura 7: submodules of the steering model

Steering actuator

The actuator is a Brushless DC servomotors and a planetary gear-head. The motor is described by a simplified static model equations

$$T_m = K_i I_m = K_i \left(\frac{V_m + k_E\omega_m}{R}\right) \quad (10)$$

where, $I_m$is the current of the motor , R is the winding resistance, k_E is the voltage constant,

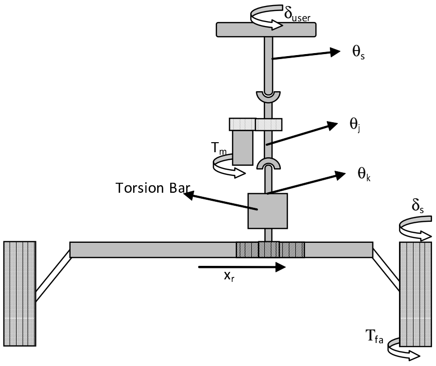

Figura 8: mechanical schematic of the Steering

K_i is the current constant and ω_m is the angular velocity of the motor.

The gear head is described by the ratio of reduction (r_g) and its efficiency (η), so the output torque and angular velocity of the gear head is given by equation (11):

$$T_j = \eta . r_g T_m \qquad \omega_m = r_g \dot{\theta}_j \tag{11}$$

Where, $\dot{\theta}_j$ is the angular velocity of the joint bar. The values of these parameters just presented are obtained from the data sheet of the motor.

Mechanical model of the steering

The schematic of the mechanical model is shown in the figure 8. Assuming that the steering bar(θ_s), the rack bar (θ_k) and the joint bar (θ_j) are rigid body(the stiffness is infinite), and that the inertia of the lower steering column (valve spool and pinion) is lumped into rack inertia and also neglecting the dynamics of the universal joint. So, the next two differential equations (12) and (13) after applying the second law of Newton for rotational motion.

$$J_1 \ddot{\theta}_j = -C_m \dot{\theta}_j - C_s \dot{\theta}_s - f_1(T_k, \theta_j) + T_m \eta r_g + \delta_{user} \tag{12}$$

$$(J_t + J_r)\ddot{\delta}_f = -C_r \dot{\delta}_f + r_s T_k - T_{fa} + T_p \tag{13}$$

where $J_1 = J_m \cdot r_g^2 + J_j + J_s$; J_m, J_j, J_s, J_t, J_r, C_m, C_s and C_r are respectively the inertias and the frictions of the motor (m), joint bar (j) and user steering column (s); r_s is the steering radius caused by the rack-pinion; T_p is the moment generated by the hydraulics system; T_k is the moment generated by the torsion bar of the hydraulic pump described by $T_k = k_t(\theta_k - r_s \delta_f)$.

Finally, the angles θ_s and θ_k are linked to the joint bar by universal joints with angles β_1e β_2 respectively(figure 9), but to simplify it will be assumed that the parameters $C_s$and $J_s$will be constant when are translated from the variable θ_s to the θ_j. So for the equation (12) it is assumed that $\theta_s = \theta_j$ and $\dot{\theta}_s = \dot{\theta}_j$. For the second joint it is possible to write the following equations (14) and (15).

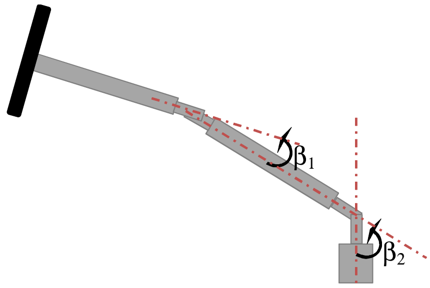

Figura 9: Angles of the universal Joints of the steering system

$$\theta_k = tan^{-1}\left(\frac{tan\theta_j}{cos\beta_2}\right) \tag{14}$$

$$f_1(T_k, \theta_j) = \left(\frac{cos\beta_2}{1 - sin^2\beta_2 cos^2\theta_j}\right) T_k \tag{15}$$

Hydraulic model of the steering

The hydraulic model is described in details in (Proca e Keyhani, 1998). The key components of the hydraulic power assisted system are the hydraulic pump, the rotary spool valve and the rack piston. The rotary spool valve consists of the torsion bar, the inner spool, and the outer sleeve (Yih, 2005).

The differential pressure inside the cylinder is proportional in non linear form to the torsion bar angle. Although, the steering torque comes from a force on the piston due to a pressure multiplied by the steering arm length, here it will be approximated linearly by $T_p \approx r_h T_k$ neglecting the dynamics of the hydraulics system.

State space model

Taking as state variables $\mathbf{x_s} = \left[\begin{array}{cccc} \theta_j & \dot{\theta}_j & \delta_f & \dot{\delta}_f \end{array}\right]^{\mathbf{T}}$ and as inputs V_m, δ_{user} and T_{fa} , we have the next state-space representation (16).

$$\dot{\mathbf{x_s}} = \left[\begin{array}{c} \dot{\theta}_j \\ \ddot{\theta}_j \\ \dot{\delta}_f \\ \ddot{\delta}_f \end{array}\right] = \left[\begin{array}{c} \dot{\theta}_j \\ \frac{1}{J_1}\left(\begin{array}{c} -\left(C_m + C_{seq}\right)\dot{\theta}_j - \left(\frac{cos\beta_2}{1-sin^2\beta_2 cos^2\theta_j}\right) T_k \ldots \\ +\eta r_g \left(K_i \frac{V_m + k_E r_g \dot{\theta}_j}{R}\right) + \delta_{user} \end{array}\right) \\ \dot{\delta}_f \\ \frac{1}{J_t + J_r}\left(-C_r \dot{\delta}_f + r_s(1 + r_h)T_k - T_{fa}\right) \end{array}\right] \tag{16}$$

where ,

$$T_k = k_t\left(tan^{-1}\left(\frac{tan\theta_j}{cos\beta_2}\right) - r_s \delta_f\right) \tag{17}$$

3 Results

3.1 Observer Design

Stochastic state-space representation

For the Kalman filter is necessary to represent the system model in a discrete stochastic state space form as represented by equation (18), which is obtained combining and discretizing the equations (8), (16), (17), (9) and the Pacejka tires equations by using the first-order Euler approximation.

$$\begin{cases} X_k = f(X_{k-1}, U_k) + w_k \\ Z_k = h(X_k) + v_k \end{cases} . \tag{18}$$

Where, the state variables are $\mathbf{X_k} = [\mathbf{x_{c,k}} : \mathbf{x_{s,k}}]$ and the input vector $\mathbf{U_k} = [v_x, \delta_{user,}, V_m]^T$. The measure vector is formed by two cases due to the update rate of the sensors. The first case $Z_{1,k}$ is formed by the angular position θ_j and the velocity $\dot{\theta}_j$ given by the encoder of the steering motor, and the yaw rate γ given by a gyroscope which is updated at 1kHz. The second case $Z_{2,k}$ is formed by the position on the plane (x_k, y_k) given by a GPS which is update at 5Hz.

Extended Kalman Filter

The EKF is a set of mathematical equations which is widely utilized(Doumiati et al., 2010). The theory is presented in (Choset, 2005) which can be described as a recursive algorithm to generate a optimal estimation in the statistical sense of the system state assuming that the system and the measuring devices has Gaussian noise which is known and that are represented by a linear model(Siegwart et al., 2011).

The algorithm of EKF was implemented using the algorithm presented in (Lai et al., 2005) which obtain a numerical approximation of the linear system of equations (18) using the theory of complex number.

3.2 Control algorithm

Taking account that the steering control system will be implemented using a servomotor that only allows implementation of a PID feedback control and feed-forward control of velocity and position, the control algorithm used was a digital PID that works at 1kHz to control the angular position of the steering motor. It can be described in linear form using the Z Transform space by the next equation:

$$V_m(z^{-1}) = \left(Kp + \frac{TK_i}{1 - z^{-1}} + \frac{K_d}{T}\left(1 - z^{-1}\right) \right) (\delta_{ref} - \theta_j)$$

where, T is the period time of the control; K_p, K_i and K_d are the proportional , integral and derivative gain calculated by tuning; and δ_{ref} is the angular position reference of the steering. The PID additionally is implemented limiting the output of V_m and with a clamping Anti-windup in the integral part.

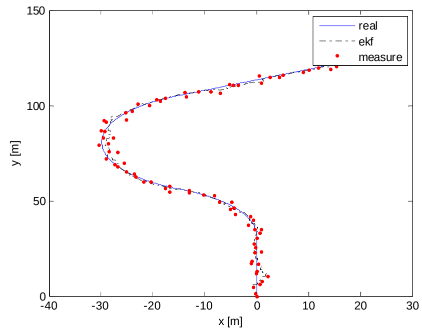

Figura 10: position of the car during the simulation

3.3 Simulations

The simulation was performed in Matlab using the ODE45 for solving the dynamic equations of the system and the script language to implement the PID and the EKF. To have some slip in the car (between $\pm 4°$) and to appreciate the estimation of the state space in challenging maneuvers it is assumed that the car velocity is $10\,m/s$ ($36\,km/h$) and the steering column reference has a repeating sequence between 0, 90°and $-90°$ as shown in figure 11. Finally the adherence between the soil and the tires has $\mu = 1$, the gaussian noise for the gyroscope has $N \sim (0, 1°/\text{s})$, the GPS has $N \sim (0, 1m)$ and the encoder has $N \sim (0, 0.1°)$.

Figure 10 shows the position of the car during the simulation, where it is possible to see the GPS response, the simulated non linear model and the estimated position of the system that decrease the ratio error in about 65% compared with the GPS. Figure 11 shows the steering angle of the vehicle where it is possible to see that the steering control response has a settling time of about 300ms and a small steady-state error caused principally by the non linear relation between the angle of the motor and the steering angle δ_f caused by the universal joint.

Although not shown by the simulation, the observability of the angular rate of the steering and the joint bar did not have a good performance. To improve this, a angular rate calculated by the motor will be used.

4 Conclusion and future works

- The mathematical model can be used to support simulation, design and implementation of control and localization algorithms.
- The universal joint between the joint bar and

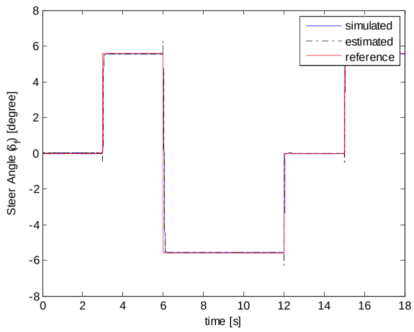

Figura 11: Steer angle of the vehicle

the hydraulic bar generate a non-linear relation between the steering output and the motor steering that should be compensated. Feedforward control is been studied and proposed in others steering by wire systems.

- The car model is in process to be simulated in GAZEBO to interact with the software which has been implemented in ROS (Harris e Conrad, 2011) and will include the exteroceptive sensors like cameras.

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