AN ALTERNATIVE METHODOLOGY FOR MODELING THE KINEMACTIS OF BIFURCATED ROBOTIC SYSTEM

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Abstract—The kinematics of an articulated robot is of extreme importance in robotics since it studies the position of its end effector and links. For the direct kinematics is used geometrical methods that depend on the robotic structure for applications. There are several methods to describe the geometry of robots, one is the well-stablished D-H notation, used successfully in cases of serial chain robots. However, this has its limitations as robotic systems become more complex (that is the case with closed chains or bifurcated). One such disadvantage relates to take this model parameters with reference to the previous link. Likewise this same model has a singularity when two consecutive axis are parallel or approximately parallel. In this case the parameters d_i of homogeneous transformation which provides the relationship between these axis are linearly dependent making the model incomplete. Due to these limitations, it was proposed the Sheth-Uicker method and the Khalil-Kleinfinger method. The aim of this paper is to extend the D-H notation creating a new reference system for the case of brachiation robot, this consisting of two arms and a body, which makes a bifurcated robot. Then as goal we have an extension of the D-H model, consisting of a recursive modification adapting it to the case of bifurcated robots in a way to make the kinematic modeling of robotic systems for easier understanding.

Keywords—Kinematics Model, Devanit-Hartenberg,Brachiation Robot.

1. Introduction

The kinematics of a robot manipulator is of extreme importance in robotics, since it studies the position of the effector and ligaments. When it comes to position, we are referring both to position itself as well as the orientation of the structure. There have been two types of kinematics, the direct kinematic where you want to find the position and velocity of the effector for a given position of the joints (this will be addressed in this work) and inverse kinematics, where we have the position and velocity of the effector and want to find the positions and velocities of the joints. For this it is necessary to use geometrical methods that depend on the robotic structure. There are several methods to describe the geometry of robots with open-chain mechanisms, one of them is the famous Denavit-Hartenberg notation used very successfully in the case of linear chains of robots[1] (Aracil, 1997), (Craig, 1955), (Rosário, 2005) though this notation has its limitations in relation to branched chain robots[2], so other methods are needed for obtaining the kinematic model in these cases. For example, it is impossible to use D-H notation in case of closed loop robots, and note even in the case of tree structure (Etienne, 1986). A solution for the tree structure cases is to use double subscripts but becomes confusing when used in robotic systems with n branches or n bifurcation. Note that there is a subtle difference between forking and branching. The tree structure robots or branched chain robots is a connection of the multiple open-loop serial robots and it has a link which connects more than two links (called a connection link) (Kanzaki, 1991). And robotic systems are bifurcated ones that we have a joint where two links out. Because of these limitations on the use of the Denavit-Hartenberg method, the Khalil-Kleinfinger method has been proposed. However the K-K model does not satisfy the problem of bifurcated robots. The aim of this paper is to extend the Denavit-Hartenberg notation for the bifurcated robots, where the main objective is to obtain a recursive model[3] to make the kinematic modelling of bifurcated robotic systems simpler.

[1]It is defined as a sequence of links and joints, where there is a link between two joints.

[2]It is derived from the serial chain and presents a structure with various.

[3]The recursive functions, which form a class of computable functions, take their name from the process of recurrence or recursion. In its most general form numerically the recursion process is to define the value of a function using other values of the same function.

2 The Devanit-Hartenberg Notation

The time course of the coordinates of the joints of a robot model represents the kinematic model of the three-dimensional system. Denavit and Hartenberg proposed translational and rotational relationships between adjacent articulated parts for mechanical manipulators. They used a matrix method, this method gives a coordinate system for each joint of the structure. Transformation matrices between these coordinate systems are designed and attached to their joints (Aracil, 1997). The D-H notation is based on the fact that to determine the relative position of two lines in space, only two parameters are needed. Therefore, if we need two parameters to define the relative position of two lines in space, then to define the relative position of two coordinate systems requires four parameters. It is interesting to note that this notation only deals with only two links together, and indeed the application of the D-H for robots with links with more than two links together is difficult and leads to ambiguities (Etienne, 1986).

The most critical problem of this notation is that it is impossible to represent branching structures simply, in this case there is the problem with the bifurcated robot composed of three links, given in Fig. 1 (Oliveira, 2008).

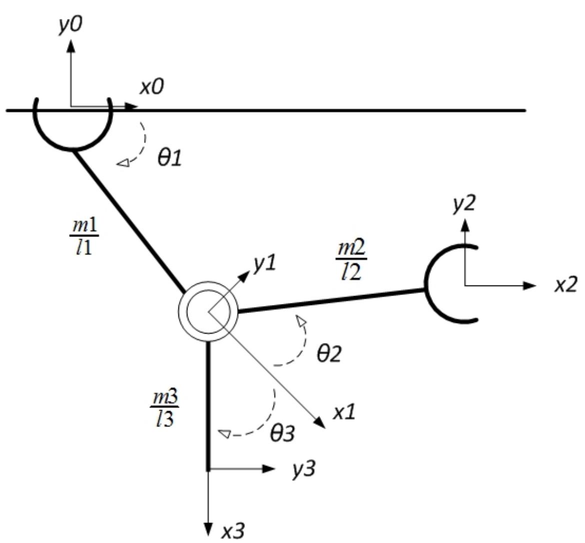

Fig. 1. Brachiation Robot.

2.1 Summary

We summarize the above procedure based on the D-H convention in the following algorithm for deriving the forward kinematics for any manipulator (Spong, 2004), (Aracil, 1997), (Craig, 1955).

- Step l. Locate and label the joint axis $z_0, \ldots, z_{n-1}$.
- Step 2. Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axis are chosen conveniently to form a right-hand frame. For $i = 1 \ldots n_{i-1}$, perform Steps 3 to 5.
- Step 3. Locate the origin O_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate O_i at this intersection. If z_i and z_{i-1} are parallel, locate O_i in any convenient position along z_i.
- Step 4. Establish x_i along the common normal between z_{i-1} and z_i throughO_i, or in the direction normal to the z_{i-1} - z_i plane if z_{i-1} and z_i intersect.
- Step 5. Establish y_i to complete a right-hand frame.
- Step 6.Establish the end-effector frame $O_0 x_0 y_0 z_0$. Assuming the n-th joint is revolute, set z_n = d along the direction z_{n-1}. Establish the origin on conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set y_n = s in the direction of the gripper closure and set $x_n = n$ as $s \times d$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-hand frame.
- Step 7. Create a table of link parameters a_i,d_i, α_i, θ_i.

2.2 The parameters of Denavit-Hartenberg

- Link Lenght (d_i) or (L_i): Distance (in modulus) measured along the common normal between axis of the joints.Translates the concept of linear separation between the axis of the joints.
- Offset or Dislocation of Joints (α_i) or (r_i): The joint dis-location reflected, in general, the distance between links along the joint as before. More in detail, is the distance (with sign) between the axis x_{i-1} and x_i measured on the axis z_{i-1} (which is common among normal x_{i-1} andx_i), starting from the $O_i$and heading toward H_i. The sign of this parameter is positive if one goes from O_{i-1} to H_i in the positive direction of z_{i-1} and negative when one walks in the opposite direction of z_{i-1}.
- Joint Angle (θ_i): Angle (with sign) defined generally between the axis of one link and of the next link. It is the angle between the axis x_{i-1} and the axis x_i, measured about the axis z_{i-1} according to the right-hand rule, i.e. the angle of rotation about the axis z_{i-1} the axis x_{i-1} must rotate so that it is parallel to the axis x_i.
- Link Twist (α_i): Torsion angle requires that the link from the axis of the former joint to the axis of the joint ahead. That is, the angle (with sign) between the axis z_{i-1} and axis z_i measured around the axis x_i, according the right-hand rule. It is nothing more

than the angle of rotation around the axis x_i the axis z_{i-1} must turn to be parallel to the axis z_i.

The four parameters are illustrates in Fig. 2

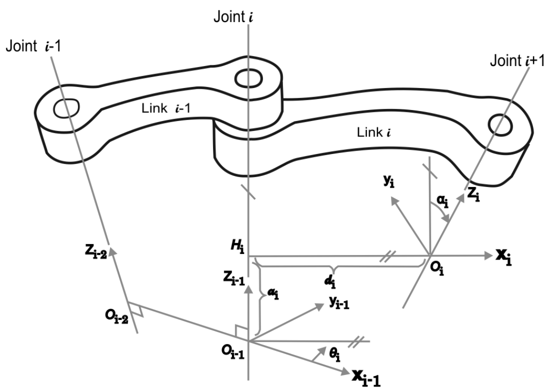

Fig. 2. D-H Notation.

The table 1 illustrates the four Denavit-Hartenberg parameters

Table 1. the four Devanit-Hartenberg parameters.

Symbol	Rotational Joint	Prismatic Joint
d_i or L_i	Fixed	Fixed
α_i or r_i	Fixed	Variable
θ_i	Variable	Fixed
α_i	Fixed	Fixed

2.3 The Kinematics Matrix of Denavit-Hartenberg

The matrix denoted by T_i^{i-1} is:

$$T_i^{i-1} = Rot(x;\ \alpha_i).Trans(z;\ r_i).Rot(z;\ \theta_i).Trans(x_i;\ L_i) \quad (1)$$

where Rot and Trans represent rotations and translations.

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i C\alpha_i & -L_i C\theta_i \\ S\theta_i & -C\theta_i C\alpha_i & -C\theta_i C\alpha_i & L_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where C and S respectively represent cos and sin.

2.4 Considerations on the Denavit-Hartenberg model

The method of D-H behaves very well with robotic systems of linear chains, so we apply the four parameters in a relatively simple manner. However, when the robotic systems become more complex if the closed chains or branched chains, the method proposed by Denavit and Hartenberg redundancy features, as described in Section Khalil-Kleinfinger. The model proposed by Denavit and Hartenberg has a singularity when two consecutive axis are parallel or approximately parallel. In this case the parameters di homogeneous transformation that provides the relationship between these axis are linearly dependent[4] Which makes the incomplete type.

3 THE KHALIL-KLEINFINGER MODEL

The aim of the new notation is to define a method that can easily be used as easy and as general Denavit-Hartenberg notation thus defining a practical method for obtaining branched kinematic model of robots.

3.1 Description (Klasing, 2009), (Etienne, 2006)

For straight chain 3D is usually used method Denavit-Hartenberg that specifies the relative position of F_i with respect to $F_{i-1}$by four parameters as specified in table I. However, it appears that the manner in which the indices are assigned this notation causes serious problems for branched kinematic structures, as will be shown in subsequent section. Soon, the four Denavit-Hartenberg parameters are used in accordance with the notation of Khalil-Kleinfinger [5]. The Fig. 3 illustrates the meaning of the four parameters khalil-Kleinfinger. The coordinate system F_i is transformed into its predecessor F_{i-1} by the following operations:

- Translation of $r_i$along the axis z_i.
- Rotation of θ_i about the axis z_i.
- Translation of d_i along the axis x_{i-1}.
- Rotation of γ_i about the axis x_{i-1}.

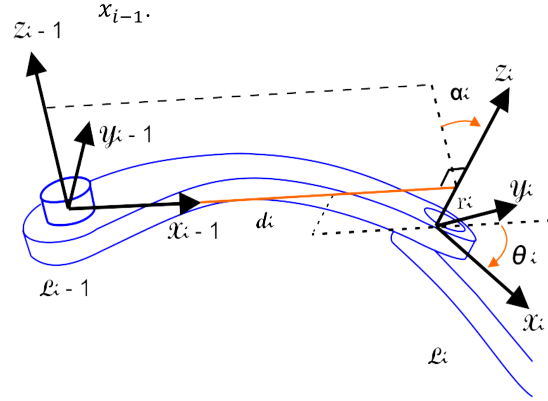

Fig. 3. khalil-Kleinfinger Parameters.

1) The Transformation Matrix: The transformation matrix of model Khalil-Kleinfinger is given by:

[4]In linear algebra, a set S of vectors is said to be linearly independent if none of its elements is a linear combination of others.

[5]For powertrains parameters Khalil-Kleinfinger θ_i,r_i,α_{i+1} and d_{i+1} correspond to the parameters of D-H.

$T_i^{i-1} = Trans(x, d_i).Rot(x, \alpha_i).Trans(u, a_{ij}).Rot(z, \theta_i).Trans(z, r_i)$ (3)

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & d_i \\ C\alpha_i S\theta_i & C\theta_i C\alpha_i & -S\alpha_i & -r_i S\alpha_i \\ S\alpha_i S\theta_i & S\alpha_i C\theta_i & C\alpha_i & r_i C\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

2) About the variable joints: A joint variable i is denoted by q_i is θ_i if i is rotational and r_i if i is prismatic. Therefore, $q_i = \theta_i(1-\sigma) + r_i\sigma_i$ where, $\sigma_i = 0$ if the joint i is rotational and $\sigma_i = 1$ if the joint i is prismatic.

3.2 Robots branched chain in Khalil-Kleinfinger Notation (Klasing, 2009), (Khalil, 2006)

The convention D-H is not suitable for this type of problem, since the contents of the specification only allow guidance of a coordinate system in relation to its predecessor. However, this problem can be remedied by taking double subscript, for example, parameters:r_{12},θ_{12}, α_{12},d_{12} describing the processing of the link $L1$ for the link $L2$ and r_{13},θ_{13}, α_{13},d_{13} describing the processing of the link $L1$ for the link $L3$ and so on. However, the use of double subscripts destroys the elegance and simplicity of D-H convention and requires the introduction of additional constraints that specify the offset orientation between the coordinate systems attached. In the case of linear orientation kinematic chains of each link L_i will be expressed by four parameters θ_i, r_i, $\alpha_{i+1}$and d_{i+1}. When a link L_i connects the more than one link L_j each additional connection L_k is described for a total of six parameters, where the four parameters r_k,θ_k, α_k,d_k specify the orientation of L_k with respect to L_i (as for the first link L_j),and two parameters γ_k and ϵ_k denoting the displacement of guidance for the first link in L_j . Again, the Z axis of each link must coincide with the axis of the joint. The axis x_i branch L_i must point to the axis z_j the first link attached L_j. For each link attached additional L_k (L_l,L_m, etc.) is helpful to imagine an auxiliary coordinate system F_i' (F_i''), F_i''', etc.) whose axis z_i' coincides with z_i but this was changed by parameter ε_k over z_i and rotated by the parameter γ_k about z_i so that the axis x_i point to the axis z_k. A Fig. 4 illustrates these parameters.

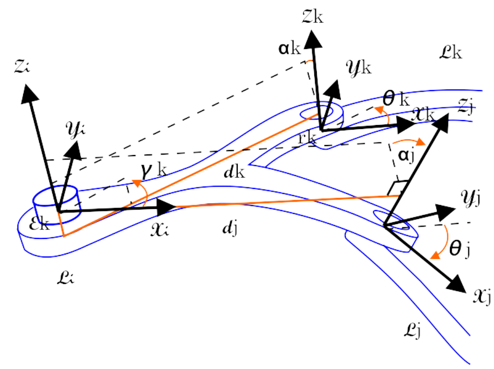

Fig. 4. Parameters of K-K for branching robots.

Observing Fig 4 is very easy to show that:

- γ_k is the angle between x_i and x_i'
- ε_k is the distance between z_i and z_k on x_i' .
- d_k is the distance between O_i' and z_k.
- θ_k is the angle between x_i' and x_k onz_k.
- r_k is the distance between O_k and x_i' .
- α_k is the angle between z_i and z_k on x_i' .

Mathematically, we have:

$$T_i^{i-1} = Rot(z_i, \gamma_j).Trans(z, \varepsilon_k) \quad (5)$$

The transformation Matriz: The points on the coordinate system of the first link attached F_j is processed in the system predecessor F_i as before using the transformation matrix given by Fig. 4. For the points system F_k a transformation is needed because of the orientation parameters r_k,θ_k,α_k and d_k are expressed relative to the auxiliary system F_i by the following matrix:

$$T_i^{i-1} = \begin{bmatrix} cos\gamma_k & -sin\gamma_k & 0 & d_i \\ sin\gamma_k & cos\gamma_k & 0 & 0 \\ 0 & 0 & 1 & \varepsilon_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Points in the system F_k, are processed in the system F_k of:

$$T_k^i = T_{i'}^i.T_j^{i'} \quad (7)$$

3.3 Example (kanzaki, 1991)

Now, we examine the khalil-Kleinfinger parameters of a robot having two arms with three joints as shown in Fig. 5 and Fig. 6. The connection table which indicates in table 2 and the link parameters of the robot are shown in the table 3-C. The chain of each branch from the terminal link to the base is as follows:

- Branch 3: link 23-link l3-branch 1-base
- Branch 2: link 22-link l2-branch 1-base
- Branch 1: link 31 -link 21-link 1-base

Each of branches can be regarded as a serial link robot.

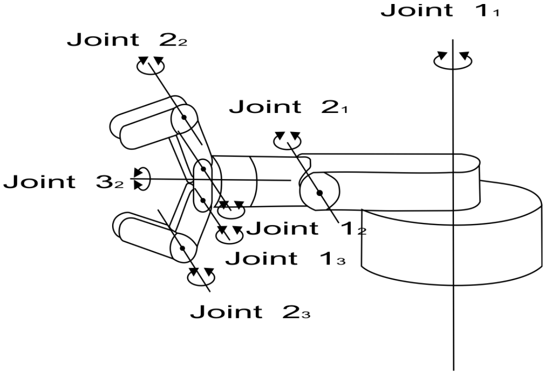

Fig. 5. A robot having two arms with three joint.

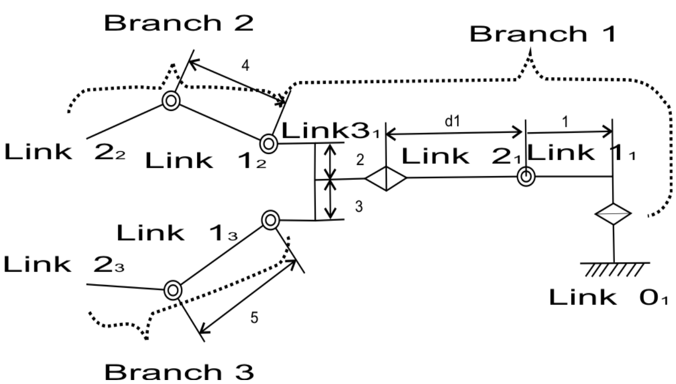

Fig. 6. Fig. Structure of a robot having two arms with three joint.

Table 2. connection table.

Branch number of base side	Branch number diverged from base side
1	2,3

Table 3. link parametes for the example.

Branch	Link i	a_i	α_i	d_i	θ_i	ϵ_i	γ_i
1	1_1	0	0^0	0	θ_{11}	0^0	0
	2_1	$l1$	90^0	0	θ_{21}	0^0	0
	3_1	0	0^0	0	θ_{31}	0^0	0
2	1_2	l_2	-90^0	0	θ_{12}	0^0	0
	2_2	l_4	0^0	0	θ_{22}	0^0	0
3	1_3	l_3	-90^0	0	θ_{13}	180^0	0^0
	2_3	l_5	0^0	0	θ_{23}	0^0	0^0

4 APPLICATION OF THE METHOD OF KHALIL-KLEINFINGER IN THE BIFURCATED ROBOT

4.1 Coordinate of links

The reference coordinate O_i=(x_i; y_i; z_i) is assigned to the link i. The system will be described using the notation of Khalil-Kleinfinger. The axis z_i is along the axis of the joint i connecting the link $i-1$ to the link i, the same can be said of the link j.The framework described is shown in Fig 7.The table 4 with the kinematic parameters is:

Table 4. Kinematic parameters for the brachiation robot.

link	$\alpha_{i,j}$	$d_{i,j}$	$\theta_{i,j}$	$r_{i,j}$
1	0	L1	θ_1	0
2	0	L2	θ_2	0
3	0	L3	θ_3	0

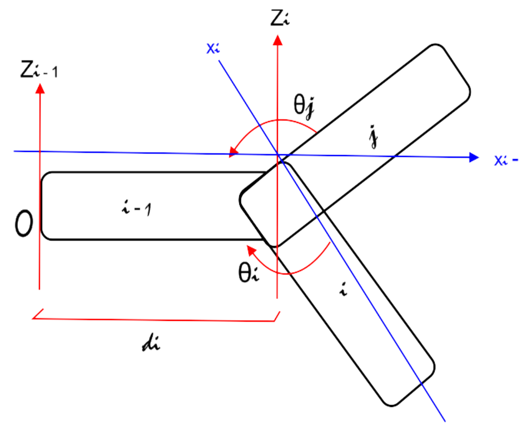

Fig. 7. Parameters K-K for the bifurcated robot of 3 links.

4.2 The Transformation Matrix

From the parameters of table IV and (4) we have:

$$T_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & L_1 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$T_i^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$T_3^0 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_3 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

which are the transformation matrices of the kinematical model for the bifurcated robot.

5 DENAVIT-HARTENBERG MODIFICATION

5.1 Indexing of Links, Joints and Frames

The links of a bifurcated robot are indexed according to the binary heap data structure (Sedgewick, 1990), very usual in the storage of binary trees.

For instance, the robot in the Fig. 8 would be indexed as shown:

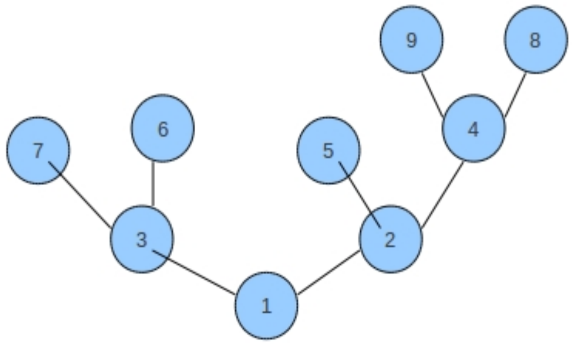

Fig. 8. Binary Tree.

We start with 1. In this setting the link before link i is the link $i = 2$, the link to the right is the link $2i$, and the link to the left is the link $2i + 1$. The link i is attached to the former link by the joint i and to the further link by the joint $2i$. The frame of a link i follows the Denavit-Hartenberg convention, except with indexes scheme of a bifurcated robot. As an example, the frame origin of the link i is in the z_{2i} axis passing through joint $2i$. Its illustrated in Fig. 9.

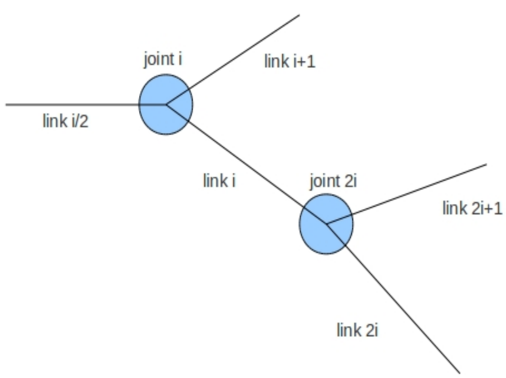

Fig. 9. Indexes for a bifurcated robot.

5.2 The Algorithm

The kinematic model of a branched robot says that each link's transformation matrix T_i should be multiplied as in a depht-first search in trees in preorder form. At each step in the recursion, the algorithm multiplies the current frame's matrix with the product of all transformation matrices already computed on the traversal.

We start at the base link i = 1:
Recursive function:

- Step 1. Let the current frame be i, if the current frame is null , return to previous frame $i/2$;
- Step 2. else, compute the frame transformation between the current link (frame i) and the product of all transformation matrices;
- Step 3. call the recursive function for the link at the right to the right using $2i$ as the current link;
- Step 4. call the recursive function for the link at the left the right using $2i + 1$ as the current link.

6 APLICATION THE DENAVIT-HARTENBERG MODIFICATION IN BIFURCATED ROBOT

Applying the modifications given by section III-C in brachiation robot and using 2 we have the following matrices:

$$T_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & L_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & L_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$T_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & L_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

$$T_1^3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & L_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

7 CONCLUSION

This paper presented models widely used in robotics. The problem of redundancy in the case of branched robots changes were made in Denavit-Hartenberg model which generated the model-Khalil Kleinfinger. Innovation takes place in changing the Denavit-Hartenberg model using the theory of tree structures (depth-first search) by changing the reference system forming a recursive algorithm.Thus one can circumvent redundancy and apply the transformation matrix of Denavit- Hartenberg. With this we can solve problems of bifurcated robots of n links via the recursive algorithm in which the parameters are given in relation to the previous setting, systematically the transformation matrices. From the studies we can still make comparisons with the Screw Theory which remains open.

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