GAIN-SCHEDULED STATE FEEDBACK CONTROL OF CONTINUOUS-TIME LPV SYSTEMS: A WHEELED MOBILE ROBOT APPLICATION

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Abstract— This paper investigates the gain-scheduled state feedback control design for continuous-time linear parameter varying (LPV) systems. It is assumed that the state-space matrices are affine functions of the time-varying parameters which belong to a known hyper-rectangle. The time-varying parameters are modeled as symmetric variables to provide new sufficient conditions for the synthesis of the gain-scheduled state feedback controller. These conditions are formulated in terms of a finite-dimensional set of linear matrix inequalities (LMI). The exponential stability of the closed-loop system with guaranteed decay rate is ensured based on the existence of a parameter-independent quadratic Lyapunov function. Numerical results presented for a trajectory tracking problem of a wheeled mobile robot with longitudinal slip illustrate the benefits of the proposed approach.

Keywords— LPV systems; Gain-scheduled control design; LMI; Mobile robot.

Resumo— Este trabalho investiga o projeto de controladores de realimentação de estados por ganho escalonado para sistemas lineares a parâmetros variantes (LPV) a tempo contínuo. Assume-se que as matrizes do espaço de estados são funções afins dos parâmetros variantes que pertencem a um hiper-retângulo conhecido. Os parâmetros variantes são modelados como variáveis simétricas para fornecer novas condições suficientes para o projeto de controladores de realimentação de estados por ganho escalonado. Essas condições são formuladas em termos de um conjunto finito de desigualdades matriciais lineares (LMI). A estabilidade exponencial do sistema em malha fechada com uma garantida taxa de decaimento é assegurada baseada na existência de uma função de Lyapunov quadrática. Resultados numéricos apresentados para um problema de rastreamento de trajetórias de um robô móvel com deslizamento longitudinal das rodas ilustram os benefícios da abordagem proposta.

Palavras-chave— Sistemas LPV; Projeto de controladores por ganho escalonado; LMI; Robôs móveis.

1 INTRODUCTION

In the past few years, linear parameter varying (LPV) systems have been intensively investigated due to the great variety of applications in engineering problems such as flight control, missile autopilots, aeroelasticity, vibroacustic control, magnetic bearings, and robotic systems. Normally, the LPV models are useful when a single linear time-invariant (LTI) model is insufficient to represent the dynamics of a plant. The LPV models can also be used to represent nonlinear plants in terms of a family of linear models (Shamma and Athans, 1991; Lawrence and Rugh, 1995; Rugh and Shamma, 2000; Paijmans et al., 2008).

In the LPV control framework, the synthesis techniques aim to ensure properties such as stability, disturbance rejection, and tracking for a family of linear time-varying (LTV) systems characterized by a linear model whose dynamics depend on time-varying parameters. For uncertain parameters, the control design consists in the synthesis of a single LTI (parameter-independent) controller that is robust to all possible parameter variations. This approach, called robust control, requires no information about the parameters besides the knowledge of its minimum and maximum values. However, the robust controllers can provide a poor performance when the time-varying parameters undergo fast and large variations (Apkarian and Gahinet, 1995; Blanchini et al., 2007; Oliveira and Peres, 2009).

Considering that the time-varying parameters can be measured, one way of reducing conservatism is to apply the gain-scheduled approach, that consists in including the parameter information in the controller design. This approach can provide higher performance due to the adjustment of the controller in real time to current operating conditions of the system. Most of the existing results for gain-scheduled control consider that the LPV system has a polytopic dependency on the parameters (Scherer, 2001; Amato et al., 2005; Lee, 2006; Blanchini et al., 2007; Dong and Guang-HongYang, 2008; De-Caigny et al., 2012).

This paper provides a new approach to the synthesis of gain-scheduled state feedback controllers for continuous-time LPV systems with affine dependency on the parameters. The synthesis problem is reduced to solving a finite-dimensional set of linear matrix inequalities (LMI) modeling the time-varying parameters as symmetric variables. Furthermore, the exponential stability of the closed-loop system with guaranteed decay rate is ensured based on the existence of parameter-independent quadratic Lyapunov function. A trajectory tracking control problem of a wheeled mobile robot with longitudinal slip illustrates the potential real-world implementations of the proposed approach.

2 PRELIMINARIES

Consider the continuous-time LPV system

$$\dot{x} = A(\theta(t))x + B(\theta(t))u, \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector and $u(t) \in \mathbb{R}^{n_u}$ is the control input. The matrices $A(\cdot) \in \mathbb{R}^{n_x \times n_x}$ and $B(\cdot) \in \mathbb{R}^{n_x \times n_u}$ are affine functions of $\theta(t)$ given in the form

$$(A(\theta(t)), B(\theta(t))) = (A_0, B_0) + \sum_{i=1}^{N} \theta_i(t)(A_i, B_i),$$

such that $\theta(t) = (\theta_1(t), \theta_2(t), \ldots, \theta_N(t))$ is the vector of exogenous non-stationary parameters that belong to the hyper-rectangle

$$\mathcal{R} := \{[\theta_{1\min}, \theta_{1\max}] \times \cdots \times [\theta_{N\min}, \theta_{N\max}]\},$$

where $\theta_{i\min}$ and $\theta_{i\max}$ are respectively the minimum and maximum values of the parameter $\theta_i(t)$.

Based on Bertsimas and Sim (2004), to each time-varying parameter $\theta_i(t)$, $i = 1, \ldots, N$, is associated a new variable $\eta_i(t) := \left(\theta_i(t) - \bar{\theta}_i\right)/\hat{\theta}_i$ that takes values in $[-1, 1]$, such that

$$\theta_i(t) = \bar{\theta}_i + \hat{\theta}_i \eta_i(t), \tag{2}$$

with $\bar{\theta}_i = \dfrac{\theta_{i\min} + \theta_{i\max}}{2}$ and $\hat{\theta}_i = \dfrac{\theta_{i\max} - \theta_{i\min}}{2}$.

3 GAIN-SCHEDULED STATE FEEDBACK CONTROL DESIGN

This section considers the design of the gain-scheduled state feedback control law

$$u = K(\theta(t))x, \quad K(\cdot) \in \mathbb{R}^{n_u \times n_x}, \tag{3}$$

for the continuous-time affine LPV system (1). The aim is to provide a finite-dimensional set of LMI conditions for the synthesis of a parameter-dependent state feedback gain $K(\cdot)$ such that the closed-loop system

$$\dot{x} = A_{c\ell}(\theta(t))x, \tag{4}$$

with

$$A_{c\ell}(\theta(t)) = A(\theta(t)) + B(\theta(t))K(\theta(t)), \tag{5}$$

is exponentially stable with a guaranteed decay rate $\alpha > 0$ for all possible trajectories of $\theta(t) \in \mathcal{R}$. According to Boyd et al. (1994), the decay rate of a continuous-time linear system is defined as the largest α such that

$$\lim_{t \to \infty} e^{\alpha t} \|x(t)\| = 0$$

holds for all nonzero solutions $x(t)$.

Based on the existence of a parameter-independent quadratic Lyapunov function, a sufficient condition to ensure exponential stability of the system (4) with a guaranteed lower bound on the decay rate is given by the following lemma.

Lemma 1 *If there exists a constant $\alpha > 0$ and a symmetric positive-definite matrix W such that*

$$\Psi(\theta(t)) := WA_{c\ell}(\theta(t))' + A_{c\ell}(\theta(t))W + 2\alpha W \leq 0, \tag{6}$$

for all $\theta(t) \in \mathcal{R}$, $t \geq 0$, then the closed-loop system (4) is exponentially stable with a guaranteed decay rate α.

The proof of Lemma 1 is a straightforward extension of the proof presented in Boyd et al. (1994) for uncertain LTI systems.

Lemma 1 consists in evaluating the parameter-dependent LMI (6) for all $\theta(t) \in \mathcal{R}$, which leads to an infinite-dimensional problem. However, a finite-dimensional set of sufficient LMI conditions can be obtained in two steps. First, the LMI (6) must be rewritten in terms of $\eta(t) = (\eta_1(t), \eta_2(t), \ldots, \eta_N(t))$, using the change of variables (2). For example:

$$A(\theta(t)) = A_0 + \sum_{i=1}^{N} \theta_i(t) A_i = A_0 + \sum_{i=1}^{N} \bar{\theta}_i A_i + \sum_{i=1}^{N} \hat{\theta}_i \eta_i(t) A_i = A(\bar{\theta}) + \sum_{i=1}^{N} \hat{\theta}_i \eta_i(t) A_i = A(\eta(t)).$$

Following this example, the LMI (6) can be represented in terms of $\eta(t)$, as follows:

$$\Psi(\eta(t)) = WA_{c\ell}(\eta(t))' + A_{c\ell}(\eta(t))W + 2\alpha W \leq 0. \tag{7}$$

Since the LMI (7) is still numerically unverifiable, the next proposition is used in the second step to obtain a finite-dimensional set of sufficient LMI conditions.

Proposition 1 *Let a matrix $\mathcal{X} \in \mathbb{R}^{n_x \times n_x}$. If there exists a matrix $\mathcal{Y} \in \mathbb{R}^{n_x \times n_x}$, such that, $\mathcal{Y} \geq \mathcal{X}$ and $\mathcal{Y} \geq -\mathcal{X}$, then, $\mathcal{Y} \geq \xi\mathcal{X}$ for all $\xi \in [-1, 1]$.*

Proof: By multiplying the inequalities $\mathcal{Y} \geq \mathcal{X}$ and $\mathcal{Y} \geq -\mathcal{X}$ by $v \in \mathbb{R}^{n_x}$ on the left and by v' on the right, it yields in the scalar inequalities $v'\mathcal{Y}v \geq v'\mathcal{X}v$ and $v'\mathcal{Y}v \geq -v'\mathcal{X}v$, respectively. Thus, $v'\mathcal{Y}v \geq |v'\mathcal{X}v| \geq \xi\, v'\mathcal{X}v$ for all $\xi \in [-1, 1]$. Consequently, $\mathcal{Y} \geq \xi\mathcal{X}$ for all $\xi \in [-1, 1]$. □

It is worth to mention that the scalar case of Proposition 1 is used in Bertsimas and Sim (2004) to provide a robust formulation for linear programming problems subject to data uncertainty. In this paper, Proposition 1 together with Lemma 1 are used to provide a sufficient condition for the existence of a parameter-dependent state feedback gain. This is the context of the next theorem.

Theorem 1 *If there exists a constant $\alpha > 0$, symmetric positive-definite matrix $W \in \mathbb{R}^{n_x \times n_x}$, matrices $V_\ell \in \mathbb{R}^{n_u \times n_x}$ and symmetric matrices $\mathcal{X}_i \in \mathbb{R}^{n_x \times n_x}$, $\mathcal{Y}_{jk} \in \mathbb{R}^{n_x \times n_x}$ and $\mathcal{Z}_i \in \mathbb{R}^{n_x \times n_x}$, for $i = 1, \ldots, N$, $j = 1, \ldots, N-1$, $k = j+1, \ldots, N$ and $\ell = 0, \ldots, N$, such that*

$$\Upsilon(\bar{\theta}) + \sum_{i=1}^{N} \left(\hat{\theta}_i \mathcal{X}_i + \hat{\theta}_i^2 \mathcal{Z}_i\right) + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \hat{\theta}_j \hat{\theta}_k \mathcal{Y}_{jk} \leq 0 \quad (8)$$

with

$$\begin{array}{rclrcl} \mathcal{X}_i & \geq & \Phi_i, & \mathcal{X}_i & \geq & -\Phi_i, \\ \mathcal{Y}_{jk} & \geq & \Gamma_{jk}, & \mathcal{Y}_{jk} & \geq & -\Gamma_{jk}, \\ \mathcal{Z}_i & \geq & \Omega_i, & \mathcal{Z}_i & \geq & -\Omega_i, \end{array} \quad (9)$$

where

$$\begin{aligned} \Upsilon(\bar{\theta}) &= WA(\bar{\theta})' + A(\bar{\theta})W + V(\bar{\theta})'B(\bar{\theta})' \\ &\quad + B(\bar{\theta})V(\bar{\theta}) + 2\alpha W, \\ \Phi_i &= WA_i' + A_iW + V_i'B(\bar{\theta})' + B(\bar{\theta})V_i \\ &\quad + V(\bar{\theta})'B_i' + B_iV(\bar{\theta}), \\ \Gamma_{jk} &= V_j'B_k' + B_kV_j + V_k'B_j' + B_jV_k, \\ \Omega_i &= V_i'B_i' + B_iV_i, \end{aligned}$$

for all $i = 1, \ldots, N$, $j = 1, \ldots, N-1$ and $k = j+1, \ldots, N$, then the parameter-dependent state feedback gain

$$K(\theta(t)) = K_0 + \sum_{i=1}^{N} \theta_i(t) K_i,$$

with $K_\ell = V_\ell W^{-1}$, for $\ell = 0, \ldots, N$, assures that the closed-loop system (4)-(5) is exponentially stable with a guaranteed decay rate α for all possible trajectories of $\theta(t) \in \mathcal{R}$.

Proof: Using the closed-loop matrix (5) and change of variables $V(\theta(t)) = K(\theta(t))W$, one has that the left-hand side of the LMI (6) is given by

$$\Psi(\theta(t)) = \Upsilon(\theta(t)).$$

Applying the change of variables (2), one has

$$\Psi(\theta(t)) = \Upsilon(\bar{\theta}) + \sum_{i=1}^{N} \left(\hat{\theta}_i \eta_i(t) \Phi_i + \hat{\theta}_i^2 \eta_i^2(t) \Omega_i\right) + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \hat{\theta}_j \hat{\theta}_k \eta_j(t) \eta_k(t) \Gamma_{jk} = \Psi(\eta(t)).$$

Defining the symmetric matrices $\mathcal{X}_i$, $\mathcal{Y}_{jk}$ and $\mathcal{Z}_i$ for $i = 1, \ldots, N$, $j = 1, \ldots, N-1$ and $k = j+1, \ldots, N$, such that (9) holds, then by Proposition 1

$$\begin{aligned} \mathcal{X}_i &\geq \eta_i(t)\Phi_i, \\ \mathcal{Y}_{jk} &\geq \eta_j(t)\eta_k(t)\Gamma_{jk}, \\ \mathcal{Z}_i &\geq \eta_i^2(t)\Omega_i, \end{aligned}$$

for all $t \geq 0$ since $\eta_i(t)$, $\eta_j(t)\eta_k(t)$ and $\eta_i^2(t)$ belong to the interval $[-1, 1]$.

Finally, one has

$$\Psi(\theta(t)) \leq \Upsilon(\bar{\theta}) + \sum_{i=1}^{N} \left(\hat{\theta}_i \mathcal{X}_i + \hat{\theta}_i^2 \mathcal{Z}_i\right) + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \hat{\theta}_j \hat{\theta}_k \mathcal{Y}_{jk}.$$

Thus, the feasibility of (8) implies the feasibility of (6). Consequently, the closed-loop system (4)-(5) is exponentially stable with a guaranteed decay rate α for all possible trajectories of $\theta(t) \in \mathcal{R}$. □

Theorem 1 provides a sufficient condition for the synthesis of a gain-scheduled state feedback controller. The parameter-dependent gain of the controller is obtained solving a feasibility test of a finite-dimensional set of LMI. The numeric complexity associated to an optimization problem formulated in terms of LMI can be estimated from the number of scalar variables and of LMI rows (Boyd et al., 1994). The LMI conditions of Theorem 1 requires checking $n_x(N^2 + 3N + 2)$ LMI rows and uses $n_x(n_x + 1)\left(N(N+3)/4 + 1/2\right) + n_x(N+1)n_u$ scalar variables.

4 WHEELED ROBOT APPLICATION

This section presents an application of the gain-scheduled control design to a trajectory tracking control problem of a wheeled mobile robot (WMR) with longitudinal slip.

4.1 Model of a Wheeled Mobile Robot

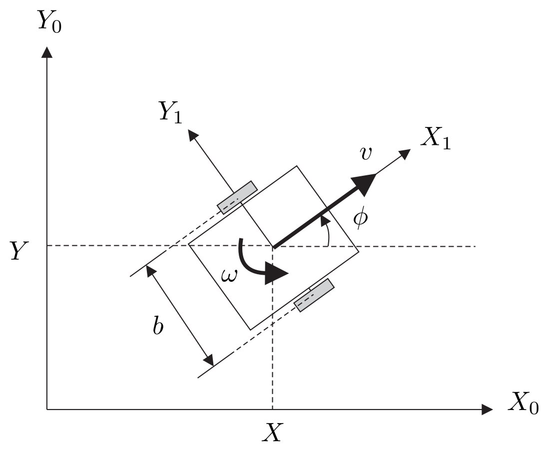

Figure 1: schematic model of a WMR.

Figure 1 shows the schematic model of a WMR. The posture of the robot, in an inertial coordinate frame (X_0, Y_0), is described by its center position $(X(t), Y(t)) \in \mathbb{R}^2$ and its orientation $\phi(t) \in \mathbb{R}$. The distance between the centerlines of the two wheels is $b > 0$. The robot translation velocity is denoted by $v(t) \in \mathbb{R}$ and its rotational velocity by $\omega(t) \in \mathbb{R}$.

Denoting the posture of the robot by $q(t) = (X, Y, \phi)' \in \mathbb{R}^3$, the kinematic model of the WMR with slip, as presented in Gonzales et al. (2009) and Iossaqui et al. (2011), is given by

$$\dot{q} = \frac{r}{2b} \begin{pmatrix} ba_l(t)\cos\phi & ba_r(t)\cos\phi \\ ba_l(t)\sin\phi & ba_r(t)\sin\phi \\ -2a_l(t) & 2a_r(t) \end{pmatrix} u \qquad (10)$$

where $r > 0$ is the radius of wheels, $u(t) = (\omega_l, \omega_r)' \in \mathbb{R}^2$ is the control input composed by the angular velocities of the left and right wheels, $\omega_l(t) \in \mathbb{R}$ and $\omega_r(t) \in \mathbb{R}$, and $0 < a_l(t) \leq 1$ and $0 < a_r(t) \leq 1$ are the longitudinal slip parameters of the left and right wheels, respectively. The slip parameter equals one means that the wheel rolls without slipping.

4.2 Trajectory Tracking Control Problem

Following Iossaqui et al. (2011), the trajectory tracking control problem consists in providing a control input $u = (\omega_l, \omega_r)'$ for the WMR such that

$$\lim_{t\to\infty} (q_{\text{ref}}(t) - q(t)) = 0,$$

where the robot posture $q(t) = (X, Y, \phi)' \in \mathbb{R}^3$ is given by (10) and the reference trajectory $q_{\text{ref}}(t) = (X_{\text{ref}}, Y_{\text{ref}}, \phi_{\text{ref}})' \in \mathbb{R}^3$ is generated using the kinematic model

$$\dot{q}_{\text{ref}} = \begin{pmatrix} \cos\phi_{\text{ref}} & 0 \\ \sin\phi_{\text{ref}} & 0 \\ 0 & 1 \end{pmatrix} u_{\text{ref}}, \qquad (11)$$

where $u_{\text{ref}}(t) = (v_{\text{ref}}(t), \omega_{\text{ref}}(t))' \in \mathbb{R}^2$ is the reference input composed by the linear and angular velocites, $v_{\text{ref}}(t) \in \mathbb{R}$ and $\omega_{\text{ref}}(t) \in \mathbb{R}$.

To ensure the robot trajectory q will follow the desired reference trajectory q_{ref}, we define the posture error $x(t) = (x_1, x_2, x_3)' \in \mathbb{R}^3$ as follows

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_{\text{ref}} - X \\ Y_{\text{ref}} - Y \\ \phi_{\text{ref}} - \phi \end{pmatrix}. \quad (12)$$

The dynamics of the posture error x, derived using (10)-(12), is given by $\dot{x} = f(t, x, u)$, with

$$f = \frac{r}{2b} \begin{pmatrix} 2g_1(t)x_2 + v_{\text{ref}}(t)\cos x_3 - bg_2(t) \\ -2g_1(t)x_1 + v_{\text{ref}}(t)\sin x_3 \\ \omega_{\text{ref}}(t) - 2g_1(t) \end{pmatrix},$$

where $g_1(t) = \omega_r/a_r(t) - \omega_l/a_l(t)$ and $g_2(t) = \omega_r/a_r(t) + \omega_l/a_l(t)$.

For the posture error to maintain equilibrium at $x = 0$, the control input u must have a steady-state component u_{ss} satisfying $f(t, 0, u_{ss}) = 0$, that results in

$$u_{ss} = \frac{1}{2ra_l(t)a_r(t)} \begin{pmatrix} a_r(t)(2v_{\text{ref}}(t) - b\omega_{\text{ref}}(t)) \\ a_l(t)(2v_{\text{ref}}(t) + b\omega_{\text{ref}}(t)) \end{pmatrix}.$$

Choosing the control input as $u = u_{ff} + u_{fb}$, with the feedforward part $u_{ff} = u_{ss}$ and the feedback part u_{fb} to be determined using the LMI thecnique proposed by Theorem 1, one has

$$\dot{x} = f(t, x, u_{fb}), \qquad (13)$$

such that $f(t, 0, 0) = 0$. Thus, following Theorem 4.15 from Khalil (2001), the exponentially stability of the origin of the nonlinear system (13) is ensured designing the stabilizing controller u_{fb} for the linearization of (13).

Linearization of (13) about the origin is given by the affine LPV system

$$\dot{x} = A(\theta(t))x + B(\theta(t))u, \qquad (14)$$

where $\theta(t) = (v_{\text{ref}}(t), \omega_{\text{ref}}(t), a_l(t), a_r(t))$,

$$A(\theta(t)) = \begin{pmatrix} 0 & \omega_{\text{ref}}(t) & 0 \\ -\omega_{\text{ref}}(t) & 0 & v_{\text{ref}}(t) \\ 0 & 0 & 0 \end{pmatrix}, \text{and}$$

$$B(\theta(t)) = \frac{r}{2b} \begin{pmatrix} -ba_l(t) & -ba_r(t) \\ 0 & 0 \\ 2a_l(t) & -2a_r(t) \end{pmatrix}.$$

Figure 2 shows the schematic representation of the closed-loop system composed of the reference trajectory, the proposed controller, and the robot.

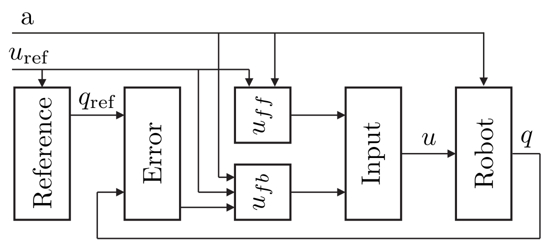

Figure 2: nonlinear closed-loop system.

4.3 Synthesis and Assessment of the Controller

This section shows the numerical results for the gain-scheduled state feedback control proposed in Section 3 applied to the trajectory tracking control problem presented in Section 4.2.

To evaluate the influence of the decay rate α on the performance of the proposed controller, it is designed two controllers, $K_{0.001}$ and $K_{0.005}$, obtained from Theorem 1 adopting $\alpha = 0.001$ and $\alpha = 0.005$, respectively. These controllers are also compared to the gain-scheduled state feedback controller K_{Mon} provided by the LMI conditions from Montagner et al. (2005, Theorem 1). The physical parameters for the model of the WMR, taken from Ryu and Agrawal (2011), are $b = 0.1624$ m and $r = 0.0825$ m. The initial conditions for the posture of the robot and the reference trajectory are $q(0) = (0, -0.5, 0)'$ and $q_{\text{ref}}(0) = (0, 0, 0)'$, respectively.

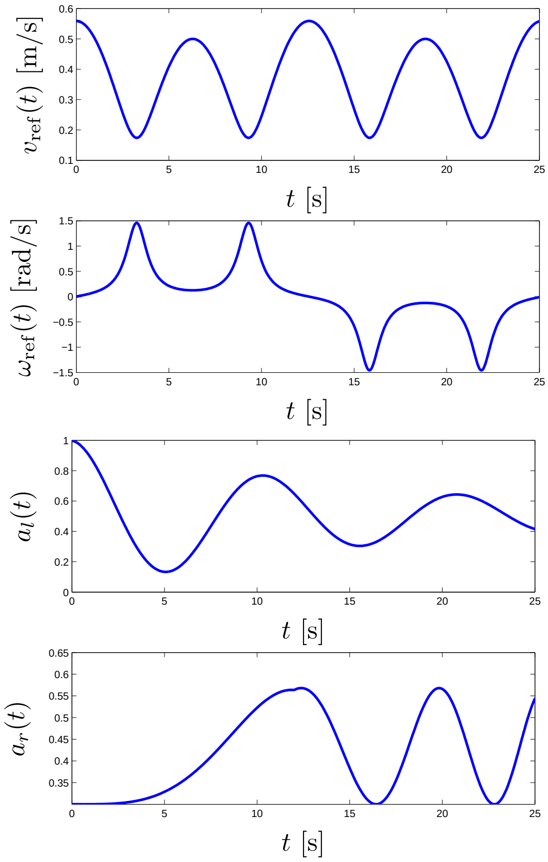

Figure 3: time-varying parameters.

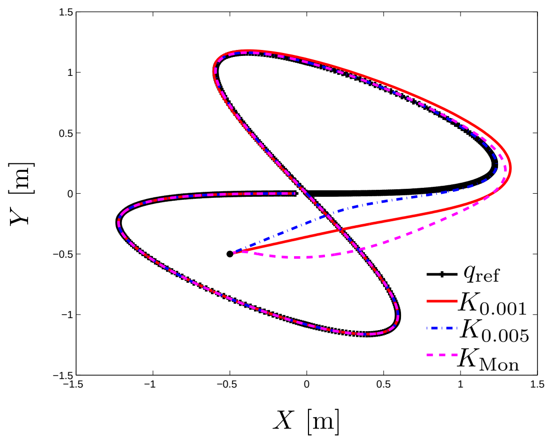

Figure 4: robot trajectory.

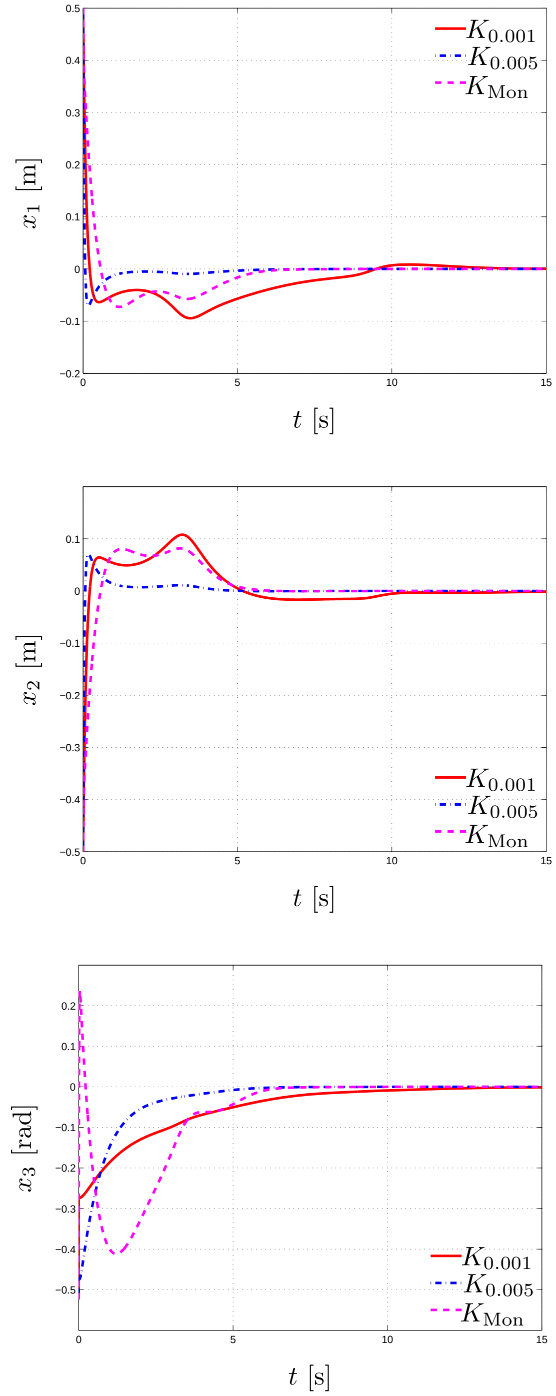

Figure 5: posture error.

Figure 3 shows the values adopted in the numerical simulation for the reference input $u_{\text{ref}}(t) = (v_{\text{ref}}(t), \omega_{\text{ref}}(t))'$ and the slip parameters $a_l(t)$ and $a_r(t)$. For the synthesis procedures, it is assumed that $v_{\text{ref}}(t) \in [0.01, 2.00 \text{ m/s}]$, $\omega_{\text{ref}}(t) \in [-\pi, \pi \text{ rad/s}]$, $a_l(t) \in [0.01, 1.00]$, and $a_r(t) \in [0.01, 1.00]$.

Figure 4 shows the reference trajectory q_{ref} and the robot trajectories q obtained using controllers $K_{0.001}$, $K_{0.005}$, and K_{Mon}. It can be seen that the robot trajectory converges to the reference trajectory.

Figure 5 shows the posture error $x = (x_1, x_2, x_3)'$. For all the designed controllers the posture error converges to zero. Controller $K_{0.001}$ provided worse results than controller K_{Mon}. However, the performance of the synthesized controllers by Theorem 1 can be further improved with larger α. As illustration, note that controller $K_{0.005}$ provided better results than both controllers $K_{0.001}$ and K_{Mon}. Finally, comparing the numerical complexity of the synthesis procedures, Theorem 1 demands 120 scalar variables and 90 LMI rows while that the conditions from Montagner et al. (2005, Theorem 1) demands 102 and 411, respectively.

5 CONCLUSIONS

New LMI conditions were presented for the synthesis of gain-scheduled state feedback controllers for continuous-time affine LPV systems based on the existence of a parameter-independent quadratic Lyapunov function. The synthesis procedure allows to choose a decay rate for the closed-loop system, thus the controller can be designed to satisfy performance specifications.

The performance of the proposed approach was demonstrated on a nonlinear trajectory tracking control problem of a WMR with longitudinal slip. It was proposed a feedforward control that allows the linearization necessary to apply the LPV technique. The synthesized controllers perform well and withstand wheel slip. For the appropriate decay rate, the proposed approach outperforms Montagner et al. (2005, Theorem 1), with less numerical complexity.

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